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211en05

LINEAR EQUATIONS

You have learnt about basic concept of a variable and a constant. You have also learnt about algebraic exprssions, polynomials and their zeroes. We come across many situations such as six added to twice a number is 20. To find the number, we have to assume the number as x and formulate a relationship through which we can find the number. We shall see that the formulation of such expression leads to an equation involving variables and constants. In this lesson, you will study about linear equations in one and two variables. You will learn how to formulate linear equations in one variable and solve them algebraically. You will also learn to solve linear equations in two variables using graphical as well as algebraic methods.

OBJECTIVES

After studying this lesson, you will be able to

- *identify linear equations from a given collection of equations;*
- *cite examples of linear equations;*
- *write a linear equation in one variable and also give its solution;*
- *cite examples and write linear equations in two variables;*
- *draw graph of a linear equation in two variables;*
- *find the solution of a linear equation in two variables;*
- *find the solution of a system of two linear equations graphically as well as algebraically;*
- *Translate real life problems in terms of linear equations in one or two variables and then solve the same.*

EXPECTED BACKGROUND KNOWLEDGE

- Concept of a variable and constant

- Algebraic expressions and operations on them
- Concept of a polynomial, zero of a polynomial and operations on polynomials

5.1 LINEAR EQUATIONS

You are already familiar with the algebraic expressions and polynomials. The value of an algebraic expression depends on the values of the variables involved it. You have also learnt about polynomial in one variable and their degrees. A polynomials in one variable whose degree is one is called a **linear polynomial** in one variable. When two expressions are separated by an equality sign, it is called an **equation**. Thus, in an equation, there is always an equality sign. The equality sign shows that the expression to the left of the sign (the left had side or LHS) is equal to the expression to the right of the sign (the right hand side or RHS). For example,

$$3x + 2 = 14 \qquad ...(1)$$

$$2y - 3 = 3y + 4 \qquad ...(2)$$

$$z^2 - 3z + 2 = 0 \qquad ...(3)$$

$$3x^2 + 2 = 1 \qquad ...(4)$$

are all equations as they contain equality sign and also contain variables. In (1), the LHS = $3x + 2$ and RHS = 14 and the variable involved is x. In (2), LHS = $2y - 3$, RHS = $3y + 4$ and both are linear polynomials in one variable. In (3) and (4), LHS is a polynomial of degree two and RHS is a number.

You can also observe that in equation (1), LHS is a polynomial of degree one and RHS is a number. In (2), both LHS and RHS are linear polynomials and in (3) and (4), LHS is a quadratic polynomial. The equations (1) and (2) are linear equations and (3) and (4) are not linear equations.

In short, an equation is a condition on a variable. The condition is that two expressions, i.e., LHS and RHS should be equal. It is to be noted that atleast one of the two expressions must contain the variable.

It should be noted that the equation $3x - 4 = 4x + 6$ is the same as $4x + 6 = 3x - 4$. Thus, an equation remains the same when the expressions on LHS and RHS are interchanged. This property is often use in solving equations.

An equation which contains two variables and the exponents of each variable is one and has no term involving product of variables is called a linear equation in two variables. For example, $2x + 3y = 4$ and $x - 2y + 2 = 3x + y + 6$ are linear equations in two variables. The equation $3x^2 + y = 5$ is not a linear equation in two variables and is of degree 2, as the exponent of the variable x is 2. Also, the equation $xy + x = 5$ is not a linear equation in two variables as it contains the term xy which is the product of two variables x and y.

The general form of a linear equation in one variable is $ax + b = 0$, $a \neq 0$, a and b are constants. The general form of a linear equation in two variables is $ax + by + c = 0$ where a, b and c are real numbers such that at atleast one of a and b is non-zero.

Example 5.1: Which of the following are linear equations in one variable? Also write their LHS and RHS.

(i) $2x + 5 = 8$

(ii) $3y - z = y + 5$

(iii) $x^2 - 2x = x + 3$

(iv) $3x - 7 = 2x + 3$

(v) $2 + 4 = 5 + 1$

Solution:

(i) It is a linear equation in x as the exponent of x is 1. LHS = $2x + 5$ and RHS = 8

(ii) It is not a linear equation in one variable as it contains two variables y and z. Here, LHS = $3y - z$ and RHS = $y + 5$

(iii) It is not a linear equation as highest exponent of x is 2. Here, LHS = $x^2 - 2x$ and RHS = $x + 3$.

(iv) It is a linear equation in x as the exponent of x in both LHS and RHS is one.
LHS = $3x - 7$, RHS = $2x + 3$

(v) It is not a linear equation as it does not contain any variable. Here LHS = $2 + 4$ and RHS = $5 + 1$.

Example 5.2: Which of the following are linear equations in two variables.

(i) $2x + z = 5$

(ii) $3y - 2 = x + 3$

(iii) $3t + 6 = t - 1$

Solution:

(i) It is a linear equation in two variables x and z.

(ii) It is a linear equation in two variables y and x.

(iii) It is not a linear equation in two variables as it contains only one variable t.

CHECK YOUR PROGRESS 5.1

1. Which of the following are linear equations in one variable?

   (i) $3x - 6 = 7$

   (ii) $2x - 1 = 3z + 2$

(iii) $5 - 4 = 1$

(iv) $y^2 = 2y - 1$

2. Which of the following are linear equations in two variables:

(i) $3y - 5 = x + 2$

(ii) $x^2 + y = 2y - 3$

(iii) $x + 5 = 2x - 3$

5.2 FORMATION OF LINEAR EQUATIONS IN ONE VARIABLE

Consider the following situations:

(i) 4 more than *x* is 11

(ii) A number *y* divided by 7 gives 2.

(iii) Reena has some apples with her. She gave 5 apples to her sister. If she is left with 3 apples, how many apples she had.

(iv) The digit at tens place of a two digit number is two times the digit at units place. If digits are reversed, the number becomes 18 less than the original number. What is the original number?

In (i), the equation can be written as $x + 4 = 11$. You can verify that $x = 7$ satisfies the equation. Thus, $x = 7$ is a solution.

In (ii), the equation is $\frac{y}{7} = 2$.

In (iii), You can assume the quantity to be found out as a variable say x, i.e., let Reena has x apples. She gave 5 apples to her sister, hence she is left with $x - 5$ apples. Hence, the required equation can be written as $x - 5 = 3$, or $x = 8$.

In (iv), Let the digit in the unit place be x. Therefore, the digit in the tens place should be 2x. Hence, the number is

$$10\,(2x) + x = 20x + x = 21x$$

When the digit are reversed, the tens place becomes x and unit place becomes 2x. Therefore, the number is $10x + 2x = 12x$. Since original number is 18 more than the new number, the equation becomes

$$21x - 12x = 18$$

or $$9x = 18$$

CHECK YOUR PROGRESS 5.2

Form a linear equation using suitable variables for the following situations:

1. Twice a number subtracted from 15 is 7.
2. A motor boat uses 0.1 litres of fuel for every kilometer. One day, it made a trip of x km. Form an equation in x, if the total consumption of fuel was 10 litres.
3. The length of rectangle is twice its width. The perimeter of rectangle is 96m. [Assume width of rectangle as y m]
4. After 15 years, Salma will be four times as old as she is now. [Assume present age of Salma as t years]

5.3 SOLUTION OF LINEAR EQUATIONS IN ONE VARIABLE

Let us consider the following linear equation in one variable,

$$x - 3 = -2$$

Here LHS = $x - 3$ and RHS = -2

Now, we evaluate RHS and LHS for some values of x

x	LHS	RHS
0	– 3	– 2
1	– 2	– 2
3	0	– 2
4	1	– 2

We observe that LHS and RHS are equal only when $x = 1$. For all other values of x, LHS $\neq$ RHS. We say that the value of x equal to 1 **satisfies** the equation or **$x = 1$ is a solution of the equation**.

A number, which when substituted for the variable in the equation makes LHS equal to RHS, is called its solution. We can find the solution of an equation by trial and error method by taking different values of the variable. However, we shall learn a systematic way to find the solution of a linear equation.

An equation can be compared with a balance for weighing, its sides are two pans and the equality symbol '=' tells us that the two pans are in balance.

We have seen the working of balance, If we put equal (and hence add) or remove equal weights, (and hence subtract) from both pans, the two pans remain in balance. Thus we can translate for an equation in the following way:

1. Add same number to both sides of the equation.
2. Subtract same number from both sides of the equation.
3. Multiply both sides of the equation by the same non-zero number.
4. Divide both sides of the equation by the same non-zero number.

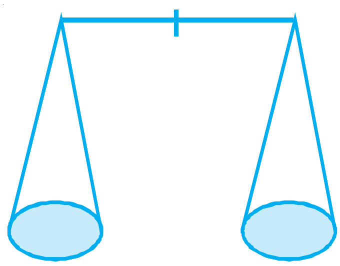

Fig 5.1

We now consider some examples:

Example 5.3: Solve $5 + x = 8$.

Solution: Subtracting 5 from both sides of the equation.

We get $\quad 5 + x - 5 = 8 - 5$

or $\quad x + 0 = 3$

or $\quad x = 3$

So, $x = 3$ is the solution of the given equation.

Check: When $x = 3$, LHS $= 5 + x = 5 + 3 = 8$ and R.H.S. $= 8$.
Therefore, LHS = RHS.

Example 5.4: Solve: $y - 2 = 7$.

Solution: Adding 2 to both sides of the equation, we get

$$y - 2 + 2 = 7 + 2$$

or $\quad y = 9$

Hence, $y = 9$ is the solution.

Check: When $y = 9$, LHS $= y - 2 = 9 - 2 = 7$ and RHS $= 7$. Therefore, LHS = RHS.

Example 5.5: Solve: $7x + 2 = 8$.

Solution: Subtracting 2 from both sides of the equation, we get

$$7x + 2 - 2 = 8 - 2$$

or $\quad 7x = 6$

or $\quad \frac{7x}{7} = \frac{6}{7}$ (dividing both sides by 7)

or $\quad x = \frac{6}{7}$

Therefore, $x = \frac{6}{7}$ is the solution of the equation.

Example 5.6: Solve: $\frac{3y}{2} - 3 = 9$

Solution: Adding 3 to both sides of the equation, we get

$$\frac{3y}{2} - 3 + 3 = 9 + 3$$

or $\frac{3y}{2} = 12$

or $\frac{3y}{2} \times 2 = 12 \times 2$ (Multiplying both sides by 2)

or $3y = 24$

or $\frac{3y}{3} = \frac{24}{3}$ (Dividing both sides by 3)

or $y = 8$

Hence, $y = 8$ is the solution.

Example 5.7: Solve the equation $2(x + 3) = 3(2x - 7)$

Solution: The equation can be written as

	$2x + 6 = 6x - 21$	
or	$6x - 21 = 2x + 6$	[Interchanging LHS and RHS]
or	$6x - 21 + 21 = 2x + 6 + 21$	[Adding 21 on both sides]
or	$6x = 2x + 27$	
or	$6x - 2x = 2x + 27 - 2x$	[Subtracting 2x from both sides]
or	$4x = 27$	
or	$x = \frac{27}{4}$	

Thus, $x = \frac{27}{4}$ is the solution of the equation.

Note:

1. It is not necessary to write the details of what we are adding, subtracting, multiplying or dividing each time.
2. The process of taking a term from LHS to RHS or RHS to LHS, is called transposing.
3. When we transpose a term from one side to other side, sign '+' changes to '–', '–' to '+'.

4. A linear equation in one variable can be written as ax + b = 0, where a and b are constants and x is the variable. Its solution is $x = -\frac{b}{a},\ a \neq 0$.

Example 5.8: Solve 3x – 5 = x +3

Solution: We have 3x – 5 = x + 3

or 3x = x + 3 + 5

or 3x – x = 8

or 2x = 8

or x = 4

Therefore, x = 4 is the solution of the given equation.

CHECK YOUR PROGRESS 5.3

Solve the following equations:

1. x – 5 = 8
2. 19 = 7 + y
3. 3z + 4 = 5z + 4
4. $\frac{1}{3}y + 9 = 12$
5. 5(x – 3) = x + 5

5.4 WORD PROBLEMS

You have learnt how to form linear equations in one variable. We will now study some applications of linear equations.

Example 5.9: The present age of Jacob's father is three times that of Jacob. After 5 years, the difference of their ages will be 30 years. Find their present ages.

Solution: Let the present age of Jacob be x years.

Therefore, the present age of his father is 3x years.

After 5 years, the age of Jacob = (x + 5) years.

After 5 years, the age of his father = (3x + 5) years.

The difference of their ages = (3x + 5) – (x + 5) years, which is given to be 30 years, therefore

$$3x + 5 - (x + 5) = 30$$

or $\quad 3x + 5 - x - 5 = 30$

or $\quad 3x - x = 30$

or $\quad 2x = 30$

or $\quad x = 15$

Therefore, the present age of Jacob is 15 years and the present age of his father = 3x = 3 × 15 = 45 years.

Check: After 5 years, age of Jacob = 15 + 5 = 20 years

After 5 years, age of his father = 45 + 5 =50 years

Difference of their ages = 50 – 20 = 30 years

Example 5.10 : The sum of three consecutive even integers is 36. Find the integers.

Solution: Let the smallest integer be x.

Therefore, other two integers are x + 2 and x + 4.

Since, their sum is 36, we have

$$x + (x + 2) + (x + 4) = 36$$

or $\quad 3x + 6 = 36$

or $\quad 3x = 36 - 6 = 30$

or $\quad x = 10$

Therefore, the required integers are 10, 12 and 14.

Example 5.11: The length of a rectangle is 3 cm more than its breadth. If its perimeter is 34 cm find its length and breadth.

Solution: Let the breadth of rectangle be x cm

Therefore, its length = x + 3

Now, since perimeter = 34 cm

We have $\quad 2(x + 3 + x) = 34$

or $\quad 2x + 6 + 2x = 34$

or $\quad 4x = 34 - 6$

or $\quad 4x = 28$

or $\quad x = 7$

Therefore, breadth = 7 cm, and length = 7 + 3 = 10 cm.

CHECK YOUR PROGRESS 5.4

1. The sum of two numbers is 85. If one number exceeds the other by 7, find the numbers.
2. The age of father is 20 years more than twice the age of the son. If sum of their ages is 65 years, find the age of the son and the father.
3. The length of a rectangle is twice its breadth. If perimeter of rectangle is 66 cm, find its length and breadth.
4. In a class, the number of boys is $\frac{2}{5}$ of the number of girls. Find the number of girls in the class, if the number of boys is 10.

5.5 LINEAR EQUATIONS IN TWO VARIABLES

Neha went to market to the purchase pencils and pens. The cost of one pencil is Rs 2 and cost of one pen is Rs 4. If she spent Rs 50, how many pencils and pens she purchased?

Since, we want to find the number of pencils and pens, let us assume that she purchased x pencils and y pens. Then,

Cost of x pencils = Rs 2 x

Cost of y pens = Rs 4 y

Since, total cost in Rs 50, we have

$$2x + 4y = 50 \qquad \ldots(1)$$

This is a linear equation in two variables x and y as it is of the form $ax + by + c = 0$

We shall now take different values of x and y to find the solution of the equation (1)

1. If $x = 1$, $y = 12$, then LHS $= 2 \times 1 + 4 \times 12 = 2 + 48 = 50$ and RHS $= 50$. Therefore, $x = 1$ and $y = 12$ is a solution.
2. If $x = 3$, $y = 11$, then LHS $= 2 \times 3 + 4 \times 11 = 50$ and RHS $= 50$. Therefore, $x = 3$, $y = 11$ is also a solution.
3. If $x = 4$, $y = 10$, then LHS $= 9 \times 4 + 4 \times 10 = 48$ and RHS $= 50$. Therefore, $x = 4$, $y = 10$ is **not** a solution of the equation.

Thus, a linear equation in two variables has more than one solution.
We have seen that a linear equation in one variable 'x' is of the form $ax + b = 0$, $a \neq 0$. It has only one solution i.e., $x = -\frac{b}{a}$. However, a linear equation in two variables x and y is of the form

$$ax + by + c = 0 \qquad ...(1)$$

where a, b and c are constants and atleast one of a or b is non-zero. Let $a \neq 0$, then (1) can be written as

$$ax = -by - c$$

or $$x = -\frac{b}{a}y - \frac{c}{a}$$

Now, for each value of y, we get a unique value of x. Thus, a linear equation in two variables will have **infinitely many solutions**.

Note: A linear equation $ax + c = 0$, $a \neq 0$, can be considered as a linear equation in two variables by expressing it as

$$ax + 0y + c = 0$$

i.e., by taking the coefficient of y as zero. It still has many solutions such as

$$x = -\frac{c}{a},\ y = 0;\ \ x = -\frac{c}{a},\ y = 1 \text{ etc.}$$

i.e., for each value of y, the value of x will be equal to $-\frac{c}{a}$.

Example 5.12: The sum of two integers is 15. Form a linear equation in two variables.

Solution: Let the two integers be x and y. Therefore, their sum = x + y. It is given that the sum is 15.

Hence, required equation is $x + y = 15$.

Example 5.13: For the equation $4x - 5y = 2$, verify whether (i) $x = 3, y = 2$ and (ii) $x = 4, y = 1$ are solutions or not.

Solution: (i) We have $4x - 5y = 2$

When $x = 3, y = 2$, LHS $= 4x - 5y = 4 \times 3 - 5 \times 2$

$= 12 - 10 = 2$

$=$ RHS

Therefore, $x = 3, y = 2$ is a solution of the given equation.

(ii) When $x = 4, y = 1$, LHS $= 4 \times 4 - 5 \times 1 = 16 - 5 = 11$

But RHS = 2. Therefore, LHS $\neq$ RHS

Hence, $x = 4, y = 1$ is not a solution.

CHECK YOUR PROGRESS 5.5

1. Form linear equations in two variables using suitable variables for the unknowns.
   (i) The perimeter of a rectangle is 98 cm. [Take length as x and breadth as y.]
   (ii) The age of father is 10 years more than twice the age of son.
   (iii) A number is 10 more than the other number.
   (iv) The cost of 2kg apples and 3 kg oranges is Rs. 120. [Take x and y as the cost per kg of apples and oranges respectively.]

Write True or False for the following:

2. $x = 0, y = 3$ is a solution of the equation
   $3x + 2y - 6 = 0$
3. $x = 2, y = 5$ is a solution of the equation
   $5x + 2y = 10$

5.6 GRAPH OF A LINEAR EQUATION IN TWO VARIABLES

You will now learn to draw the graph of a linear equation in two variables. Consider the equation $2x + 3y = 12$. It can be written as

$$2x = 12 - 3y \text{ or } 3y = 12 - 2x$$

$$x = \frac{12 - 3y}{2} \quad \text{or} \quad y = \frac{12 - 2x}{3}$$

Now, for each value of y or for each value of x, we get a unique corresponding value of x or y. We make the following table for the values of x and y which satisfy the equation:

$2x + 3y = 12$

x	0	6	3	9	– 3
y	4	0	2	– 2	6

Thus, $x = 0, y = 4$; $x = 6, y = 0$; $x = 3, y = 2$; $x = 9, y = -2$; $x = -3, y = 6$ are all solutions of the given equation.

We write these solutions as order pairs (0, 4), (6, 0), (3, 2), (9, – 2) and (– 3, 6).

Here, first entry gives the value of x and the corresponding second entry gives the value of y. We will now learn to draw the graph of this equation by plotting these ordered pairs in a plane and then join them. In the graph of $2x + 3y = 12$, the points representing the solutions will be on a line and a point which is not a solution, will not lie on this line. Each point also called orderd pair, which lies on the line will give a solution and a point which does not lie on the line will not be a solution of the equation.

To draw the graph of a linear equation in two variables, we will first plot these points in a plane. We proceed as follows:

Step 1: We take two perpendicular lines X'OX and YOY' intersecting at O. Mark the real numbers on X'OX and YOY' by considering them as number lines with the point O as the real number 0 as shown in Fig 5.2. These two lines divide the plane into four parts, called first quadrant, second quadrant, third quadrant and fourth quadrant. The number line X'OX is called **x-axis** and the line Y'OY is called **y-axis**. Since, we have taken x-axis and y-axis, perpendicular to each other in a plane, we call the plane as coordinate plane or **cartesian plane** in the honour of French mathematician Descartes who invented this system to plot a point in the plane.

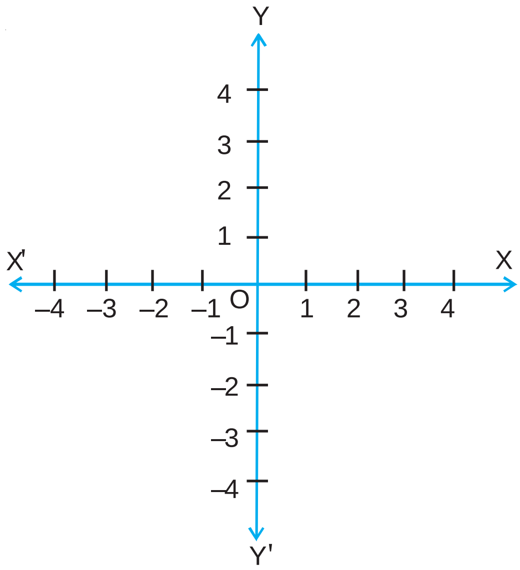

Fig 5.2

Step 2: To plot a point say (3, 2), take the point 3 on x-axis and through this point, draw a line '*l*' perpendicular to x-axis (i.e. parallel to y-axis). Now take the point 2 on y-axis and through 2, draw a line 'm' perpendicular to y-axis (i.e. parallel to x-axis) to meet *l* at P. The point P represents the point (3, 2) on the plane.

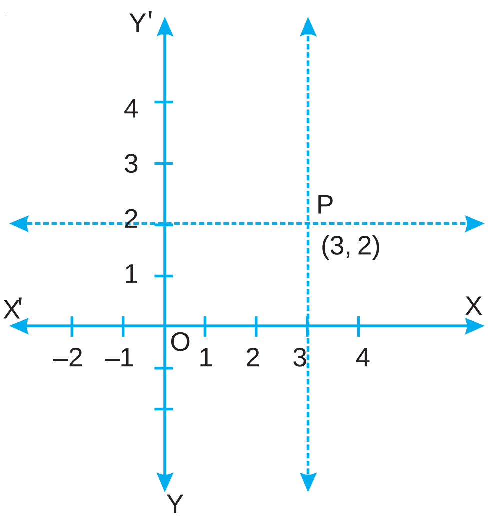

Fig. 5.3

Note 1: It may be noted that, for the ordered pair (a, b), a is called **x-coordinate** and b is called **y-coordinate**.

Note 2: Every point on x-axis can be written as (a, 0) i.e. its y-coordinate is zero and every point on y-axis is of the form (0, b) i.e., its x-coordinate is zero. The coordinates of the point O are (0, 0).

Note 3: In the first quadrant, both x and y coordinates are positive, in the second quadrant, x coordinate is negative and y coordinate is positive, in the third quadrant both x and y coordinates are negative and in the fourth quadrant, x-coordinate is positive and y-coordinate is negative.

Example 5.14: Represent the point (–2, 3) in the coordinate plane.

Solution: Draw x-axis and y-axis on the plane and mark the points on them. Take the point –2 on x-axis and draw the line *l* parallel to y-axis. Now take the point 3 on y-axis and draw the line 'm' parallel to x-axis to meet *l* at P. The point P represent (–2, 3), we say (–2 , 3) are coordinates of the point P.

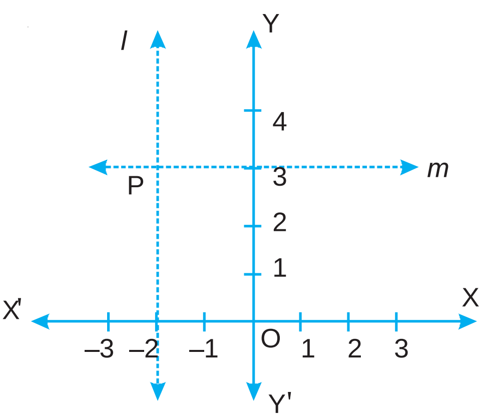

Fig 5.4

You will now learn to draw the graph of a linear equation in two variables. It should be noted that the graph of linear equation in two variables is a line and the coordinates of every point on the line satisfies the equation. If a point does not lie on the graph then its coordinates will not safisfy the equation. You also know that from two given points, one and only one line can be drawn. Therefore, it is sufficient to take any two points, i.e., values of the variables x and y which satisfy the equation. However, it is suggested that you should take three points to avoid any chance of a mistake occurring.

Example 5.15: Draw the graph of the equation 2x – 3y = 6.

Solution: Now choose values of x and y which satisfy the equation 2x – 3y = 6. It will be easy to write the equation by transforming it in any of the following form

$$2x = 3y + 6 \text{ or } 3y = 2x - 6$$

$$\Rightarrow\ x = \frac{3y+6}{2} \ \text{ or } \ y = \frac{2x-6}{3}$$

Now by taking different values of x or y, you find the corresponding values of y or x. If we take different values of x in $y = \frac{2x-6}{3}$, we get corresponding values of y. If x = 0, we get y = – 2, x = 3 gives y = 0 and x = – 3 gives y = – 4.

You can represent these values in the following tabular form:

x	0	3	–3
y	– 2	0	– 4

The corresponding points in the plane are (0, –2), (3, 0) and (–3, –4). You can now plot these points and join them to get the line which represents the graph of the linear equation as shown here.

Note that all the three points must lie on the line.

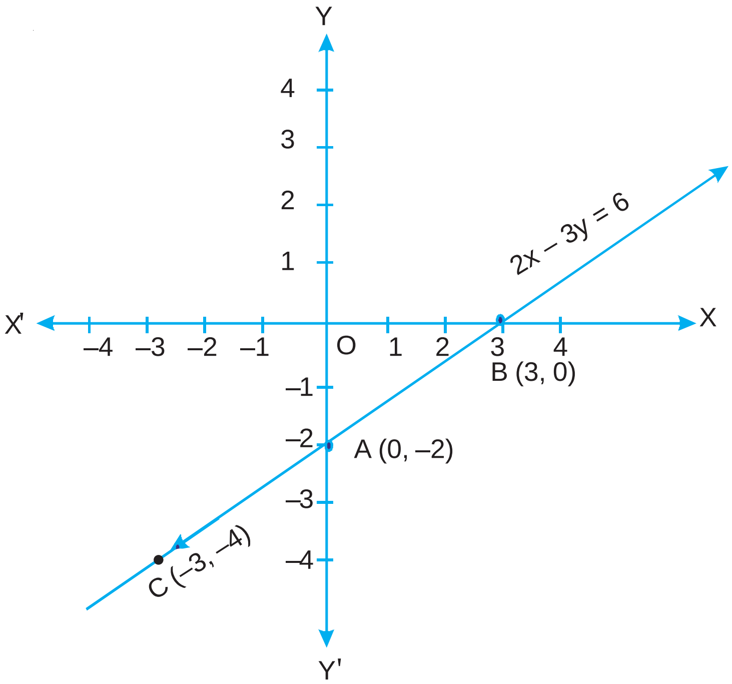

Fig 5.5

Example 5.16: Draw the graph of the equation x = 3.

Solution: It appears that it is a linear equation in one variable x. You can easily convert it into linear equation in two variables by writing it as

$$x + 0\,y = 3$$

Now you can have the following table for values of x and y.

x	3	3	3
y	3	0	1

Observe that for each value of y, the value of x is always 3. Thus, required points can be taken as (3, 3), (3, 0), (3, 1). The graph is shown in Fig. 5.6.

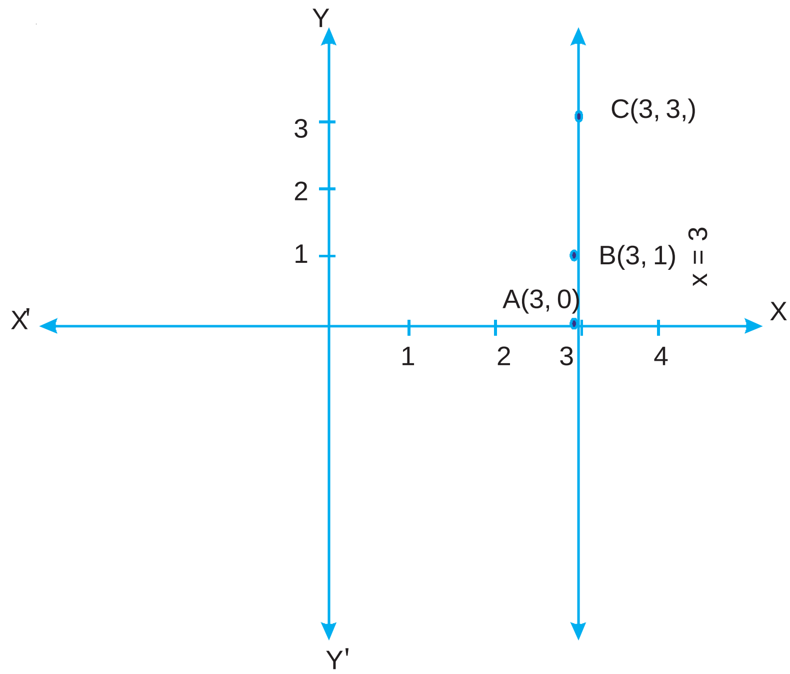

Fig. 5.6

CHECK YOUR PROGRESS 5.6

1. Plot the following points in the cartesian plane:

   (i) (3, 4) (ii) (–3, –2) (iii) (–2, 1)

   (iv) (2, –3) (v) (4, 0) (vi) (0, –3)

2. Draw the graph of each of the following linear equations in two variables:

   (i) $x + y = 5$ (ii) $3x + 2y = 6$

   (iii) $2x + y = 6$ (iv) $5x + 3y = 4$

5.7 SYSTEM OF LINEAR EQUATIONS IN TWO VARIABLES

Neha went to market and purchased 2 pencils and 3 pens for ₹ 19. Mary purchased 3 pencils and 2 pens for ₹ 16. What is the cost of 1 pencil and 1 pen? If the cost of one pencil is ₹ x and cost of one pen is ₹ y, then the linear equation in case of Neha is $2x + 3y = 19$ and for Mary it is $3x + 2y = 16$. To find the cost of 1 pencil and 1 pen, you have to find those values of x and y which satisfy both the equations, i.e.,

$$2x + 3y = 19$$

$$3x + 2y = 16$$

These two equations taken together are called system of linear equations in two variables and the values of x and y which satisfy both equations simultaneously is called the solution.

There are different methods for solving such equation. These are graphical method and algebraic method. You will first learn about graphical method and then algebraic method for solving such equations.

5.7.1 Graphical method

In this method, you have to draw the graphs of both linear equations on the same graph sheet. The graphs of the equations may be

(i) **Intersecting lines:** In this case, the point of intersection will be common solution of both simultaneous equations. The x-coordinate will give the value of x and y-coordinate will given value of y. In this case system will have a unique solution.

(ii) **Concident lines:** In this case each point on the common line will give the solution. Hence, system of equations will have infinitely many solutions.

(iii) **Parallel lines:** In this case, no point will be common to both equations. Hence, system of equations will have no solution.

Example 5.17: Solve the following system of equations:

$$x - 2y = 0 \qquad ...(1)$$

$$3x + 4y = 20 \qquad ...(2)$$

Solution: Let us draw the graphs of these equations. For this, you need atleast two solutions of each equation. We give these values in the following tables.

$x - 2y = 0$

x	0	2	–2
y	0	1	–1

$3x + 4y = 20$

x	0	4	6
y	5	2	1/2

Now plot these points on the same graph sheet as given below:

The two graphs intersect at the point P whose coordinates are (4, 2). Thus $x = 4$, $y = 2$ is the solution.

You can verify that $x = 4$, $y = 2$ satisfies both the equations.

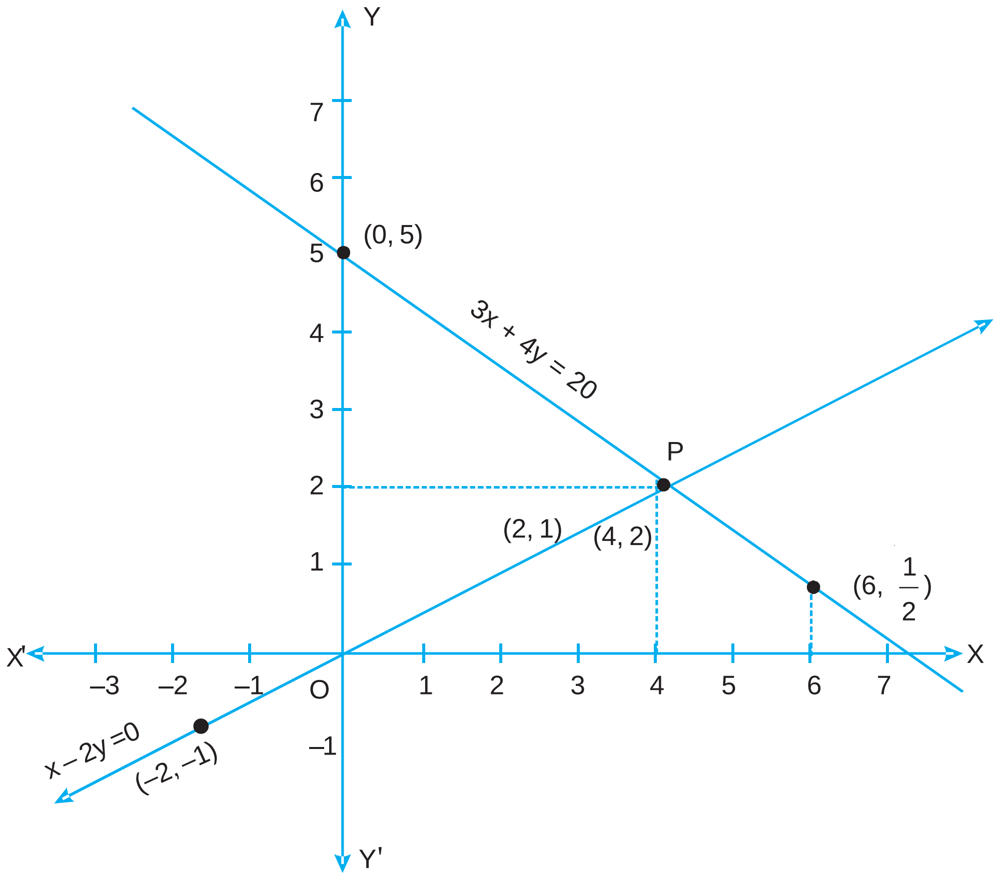

Fig. 5.7

Example 5.18: Solve the following system of equations:

$$x + y = 8 \qquad \ldots(1)$$

$$2x - y = 1 \qquad \ldots(2)$$

Solution: To draw the graph of these equation, make the following by selecting some solutions of each of the equation.

$x + y = 8$

x	3	4	5
y	5	4	3

$2x - y = 1$

x	0	1	2
y	–1	1	3

Now, plot the points (3, 5), (4, 4) and (5, 3) to get the graph of $x + y = 8$ and (0, –1), (1, 1) and (2, 3) to get the graph of $2x - y = 1$ on the same graph sheet. The two lines intersect at the point P whose coordinates are (3, 5). Thus $x = 3$, $y = 5$, is the solution of the system of equations. You can verify that $x = 3$, $y = 5$ satisfies both equations simultaneously.

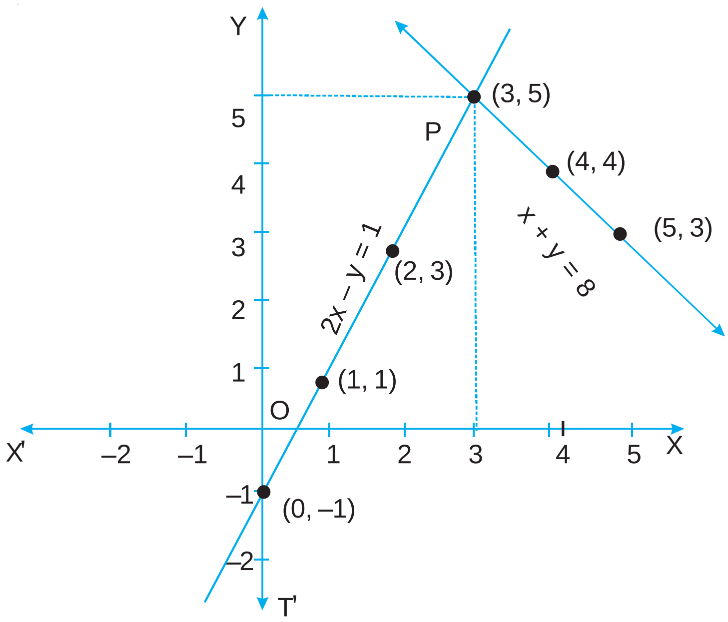

Fig. 5.8

Example 5.19: Solve the following system of equations:

$$x + y = 2 \qquad \text{...(1)}$$

$$2x + 2y = 4 \qquad \text{...(2)}$$

Solution: First make tables for some solutions of each of the equation.

$x + y = 2$

x	0	2	1
y	2	0	1

$2x + 2y = 4$

x	0	2	1
y	2	0	1

Now draw the graph of these equations by plotting the corresponding points.

You can see that graph of both the equations is the same. Hence, system of equations has infinitely many solutions. For example, $x = 0, y = 2$; $x = 1, y = 1$; $x = 2, y = 0$ etc. You can also observe that these two equations are essentially the same equation.

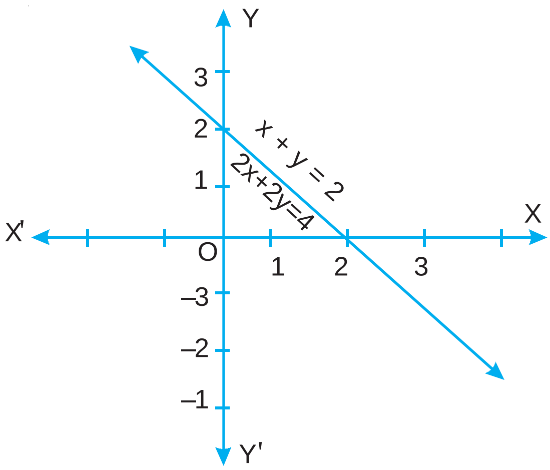

Fig. 5.9

Example 5.20: Solve the following system of equations:

$$2x - y = 4 \qquad ...(1)$$

$$4x - 2y = 6 \qquad ...(2)$$

Solution: Let us draw the graph of both equations by taking some solutions of each of the equation.

$2x - y = 4$

x	0	2	–1
y	–4	0	–6

$4x - 2y = 6$

x	0	1.5	2
y	–3	0	1

You can observe that these graphs are parallel lines. Since, they do not have any common point, the system of equations, therefore, has no solution.

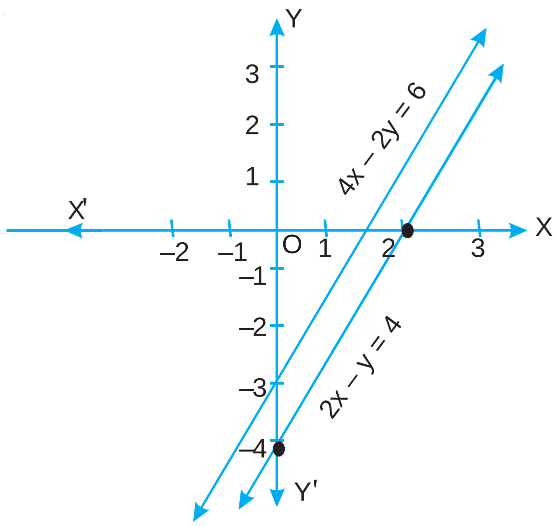

Fig. 5.10

CHECK YOUR PROGRESS 5.7

Solve the following system of equations graphically. Also, tell whether these have unique solution, infinitely many solutions or no solution.

1. $x - y = 3$

   $x + y = 5$

2. $2x + 3y = 1$

   $3x - y = 7$

3. $x + 2y = 6$

   $2x + 4y = 12$

4. $3x + 2y = 6$

   $6x + 4y = 18$

5. $2x + y = 5$

   $3x + 2y = 8$

5.7.2 Algebraic Method

There are several methods of solving system of two linear equations in two variables. You have learnt one method which is known as graphical method. We shall now discuss here two more methods, called algebraic methods. They are

(i) Substitution Method.

(ii) Elimination method.

Note: These methods are useful in case the system of equations has a unique solution.

Substitution Method: In this method, we find the value of one of the variable from one equation and substitute it in the second equation. This way, the second equation will be reduced to linear equaion in one variable which we have already solved. We explain this method through some examples.

Example 5.21: Solve the following system of equations by substitution method.

$$5x + 2y = 8 \qquad \ldots(1)$$

$$3x - 5y = 11 \qquad \ldots(2)$$

Solution: From (1), we get

$$2y = 8 - 5x$$

or $$y = \frac{1}{2}(8 - 5x) \qquad \ldots(3)$$

Substituting the value of y in (2), we get

$$3x - \frac{5}{2}(8 - 5x) = 11$$

or $$6x - 5(8 - 5x) = 22 \qquad \text{[multiplying both sides by 2]}$$

or $$6x - 40 + 25x = 22$$

or $$31x = 40 + 22$$

or $$x = \frac{62}{31} = 2$$

Substituting the value of x = 2 in (3), we get

$$y = \frac{1}{2}(8 - 5\times 2) = \frac{1}{2}(8 - 10)$$

or $$y = -\frac{2}{2} = -1$$

So, the solution to the system of equations is $x = 2, y = -1$.

Example 5.22: Solve the following system of equations by substitution method:

$$2x + 3y = 7 \qquad ...(1)$$

$$3x + y = 14 \qquad ...(2)$$

Solution: From equation (2), we get

$$y = 14 - 3x \qquad ...(3)$$

Substituting the value of y in (1), we get

$$2x + 3(14 - 3x) = 7$$

or $$2x + 42 - 9x = 7$$

or $$2x - 9x = 7 - 42$$

or $$-7x = -35$$

Therefore $$x = \frac{-35}{-7} = 5$$

Substituting the value of x in (3), we get

$$y = 14 - 3x = 14 - 3 \times 5$$

or $$y = 14 - 15 = -1$$

Hence, $x = 5, y = -1$ is the solution.

Check: You can verify that $x = 5, y = -1$ satisfies both the equations.

CHECK YOUR PROGRESS 5.8

Solve the following system of equations by substitution method:

1. $x + y = 14$
   $x - y = 2$

2. $2x + 3y = 11$
   $2x - 4y = -24$

3. $3x + 2y = 11$
   $2x + 3y = 4$

4. $7x - 2y = 1$
   $3x + 4y = 15$

Elimination Method: In this method, we eliminate one of the variable by multiplying both equations by suitable non-zero contants to make the coefficients of one of the variable numerically equal. Then we add or subtract one equation to or from the other so that one variable gets eliminated and we get an equation in one variable. We now consider some examples to illustrate this method.

Example 5.23: Solve the following system of equations using elimination method.

$$3x - 5y = 4 \quad \ldots(1)$$

$$9x - 2y = 7 \quad \ldots(2)$$

Solution: To elminate x, multiply equation (1) by 3 to make coefficient of x equal. You get the equations.

$$9x - 15y = 12 \quad \ldots(3)$$

$$9x - 2y = 7 \quad \ldots(4)$$

Subtracting (4) from (3), we get

$$9x - 15y - (9x - 2y) = 12 - 7$$

or $\quad 9x - 15y - 9x + 2y = 5$

or $\quad -13\, y = 5$

or $\quad y = -\dfrac{5}{13}$

Substituting $y = -\dfrac{5}{13}$ in equation (1), we get

$$3x - 5 \times \left(-\frac{5}{13}\right) = 4$$

or $\quad 3x + \dfrac{25}{13} = 4$

or $\quad 3x = 4 - \dfrac{25}{13} = \dfrac{27}{13}$

or $\quad x = \dfrac{9}{13}$

Therefore, $x = \frac{9}{13}$ and $y = -\frac{5}{13}$ is the required solution of the given system of equations.

Example 5.24: Solve the following system of equations using elimination method.

$$2x + 3y = 13 \qquad ...(1)$$

$$5x - 7y = -11 \qquad ...(2)$$

Solution: To eliminate y, multiply equation (1) by 7 and equation (2) by 3, we get

$$14x + 21y = 91 \qquad ...(3)$$

$$15x - 21y = -33 \qquad ...(4)$$

Adding (3) and (4), we get

$$29\,x = 58$$

or $\quad x = \frac{58}{29} = 2$

Substituting x = 2 in (1), we get

$$2 \times 2 + 3y = 13$$

or $\quad 3y = 13 - 4 = 9$

or $\quad y = \frac{9}{3} = 3$

Therefore, x = 2 and y = 3 is the solution of the given system of equations.

CHECK YOUR PROGRESS 5.9

Solve the following systems of equations by elimination method:

1. $3x + 4y = -6$
   $3x - y = 9$
2. $x + 2y = 5$
   $2x + 3y = 8$
3. $x - 2y = 7$
   $3x + y = 35$
4. $3x + 4y = 15$
   $7x - 2y = 1$
5. $2x + 3y = 4$
   $3x + 2y = 11$
6. $3x - 5y = 23$
   $2x - 4y = 16$

5.8 WORD PROBLEMS

Example 5.25: The perimeter of a rectangular garden is 20 m. If the length is 4 m more than the breadth, find the length and breadth of the garden.

Solution: Let the length of the garden be x m. Therefore, breadth of garden = (x – 4) m. Since, perimeter is 20 m, so

$$2\,[x + (x - 4)] = 20$$

or $\quad 2(2x - 4) = 20$

or $\quad 2x - 4 = 10$

or $\quad 2x = 10 + 4 = 14$

or $\quad x = 7$

Hence, length = 7 m and breadth = 7 – 4 = 3 m.

Alternatively, you can solve the problem using two variables. Proceed as follows:

Let the length of garden = x m

and width of garden = y m

Therefore $\quad x = y + 4 \qquad \ldots(1)$

Also, perimeter is 20 m, therefore

$$2(x + y) = 20$$

or $\quad x + y = 10 \qquad \ldots(2)$

Solving (1) and (2), we get x = 7, y = 3

Hence, length = 7 m and breadth = 3m

Example 5.26: Asha is five years older than Robert. Five years ago, Asha was twice as old as Robert was then. Find their present ages.

Solution: Let present age of Asha be x years

and present age of Robert be y years

Therefore, $\quad x = y + 5$

or $\quad x - y = 5 \qquad \ldots(1)$

5 years ago, Asha was x – 5 years and Robert was (y – 5) years old.

Therefore, $\quad x - 5 = 2(y - 5)$

or $\quad x - 2y = -5 \qquad \ldots(2)$

Solving (1) and (2), we get y = 10 and x = 15

Hence, present age of Asha = 15 years and present age of Robert = 10 years.

Example 5.27: Two places A and B are 100 km apart. One car starts from A and another from B at the same time. If they travel in the same direction, they meet after 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of the two cars. Assume that the speed of car at A is more than the speed of car at B.

Solution: Let speed of the car starting from A be x km/h

and speed of the car starting from B be y km/h

Therefore, the distance travelled by car at A in 5 hours = 5x km

and the distance travelled by car at B in 5 hours = 5y km

Since they meet after 5 hours when they travel in the same direction, the car at A has travelled 100 km more than the car at B. Therefore,

$$5x - 5y = 100$$

or $$x - y = 20 \quad ...(1)$$

When they travel towards each other, they meet after 1 hour. It means, total distance travelled by car at A and car at B in 1 hour is 100 km

Therefore $$x + y = 100 \quad ...(2)$$

Solving (1) and (2), we get $x = 60$ and $y = 40$

Therefore, the speed of car at A = 60 km/h and

the speed of car at B = 40 km/h.

CHECK YOUR PROGRESS 5.10

1. Rahim's father is three times as old as Rahim. If sum of their ages is 56 years, find their ages.
2. Rita has 18m of cloth. She cut it into two pieces in such a way that one piece is 4 m longer than the other. What is the length of shorter piece.
3. A total of Rs 50000 is to be distributed among 200 persons as prizes. A prize is either Rs 500 or Rs 100. Find the number of each type of prizes.
4. A purse contain Rs 2500 in notes of denominations of 100 and 50. If the number of 100 rupee notes is one more than that of 50 rupee notes, find the number of notes of each denomination.

LET US SUM UP

- An equation in one variable of degree one is called a linear equation in variable.
- The general form of a linear equation in one variable is $ax + b = 0$, $a \neq 0$, a and b are real numbers.
- The value of the variable which satisfies the linear equation is called its solution or root.
- To solve a word problem, it is first translated into algebriac statements and then solved.
- The general form of a linear equation in two variables is $ax + by + c = 0$, where a, b, c are real numbers and atleast one of a or b is non zero.
- The equation $ax + c = 0$ can be expressed as linear equation in two variables as $ax + 0y + c = 0$.
- To draw the graph of a linear equation in two variables, we find atleast two points in plane whose coordinates are solutions of the equation and plot them.
- The graph of a linear equaion in two variables is a line.
- To solve two simultaneous equations in two variables, we draw their graphs on the same graph paper.

  (i) if graph is intersecting lines, point of intersection gives unique solution.

  (ii) If graph is the same line, system has infinitely many solutions

  (iii) If graph is parallel lines, system of equation has no solution
- Algebraic methods of solving system of linear equations are

  (i) Substitution method

  (ii) Elimination method
- To solve word problems, we tranlate the given information (data) into linear equations and solve them.

TERMINAL EXERCISE

1. Choose the correct option:

   (i) Which one of the following is a linear equation in one variable?

   (A) $2x + 1 = y - 3$ (B) $3t - 1 = 2t + 5$

   (C) $2x - 1 = x^2$ (D) $x^2 - x + 1 = 0$

   (ii) Which one of the following is not a linear equation?

   (A) $5 + 4x = y + 3$ (B) $x + 2y = y - x$

(C) $3 - x = y^2 + 4$ (D) $x + y = 0$

(iii) Which of the following numbers is the solution of the equation $2(x + 3) = 18$?

(A) 6 (B) 12

(C) 13 (D) 21

(iv) The value of x, for which the equation $2x - (4 - x) = 5 - x$ is satisfied, is:

(A) 4.5 (B) 3

(C) 2.25 (D) 0.5

(v) The equation $x - 4y = 5$ has

(A) no solution (B) unique solution

(C) two solutions (D) infinitely many solutions

2. Solve each of the following equations

(i) $2z + 5 = 15$ (ii) $\frac{x+2}{3} = -2$

(iii) $\frac{4-2y}{3} + \frac{y+1}{2} = 1$ (iv) $2.5x - 3 = 0.5x + 1$

3. A certain number increased by 8 equals 26. Find the number.

4. Present ages of Reena and Meena are in the ration 4 : 5. After 8 years, the ratio of their ages will be 5 : 6. Find their present ages.

5. The denominator of a rational number is greater than its numerator by 8. If the denominaor is decreased by 1 and numerator is increased by 17, the number obtained is $\frac{3}{2}$. Find the rational number

6. Solve the following system of equations graphically:

(i) $x - 2y = 7$, $x + y = -2$ (ii) $4x + 3y = 24$, $3y - 2x = 6$

(iii) $x + 3y = 6$, $2x - y = 5$ (iv) $2x - y = 1$, $x + y = 8$

7. Solve the following system of equations :

(i) $x + 2y - 3 = 0$, $x - 2y + 1 = 0$ (ii) $2x + 3y = 3$, $3x + 2y = 2$

(iii) $3x - y = 7$, $4x - 5y = 2$ (iv) $5x - 2y = -7$, $2x + 3y = -18$

8. The sum of the digits of a two-digit number is 11. If the digits are reversed, the new number is 27 less than the original number. Find the original number.

9. Three years ago Atul's age was four times Parul's age. After 5 years from now, Atul's age will be two times Parul's age. Find their present ages.

10. The perimeter of a rectangular plot of land is 32 m. If the length is increased by 2m and breadth is decreased by 1 m, the area of the plot remains the same. Find the length and breadth of the plot.

ANSWERS TO CHECK YOUR PROGRESS

5.1

1. (i) 2. (i)

5.2

1. $15 - 2x = 7$
2. $0.1x = 10$
3. $6y = 96$
4. $t + 15 = 4t$

5.3

1. $x = 13$ 2. $y = 12$ 3. $z = 0$

4. $y = 9$ 5. $x = 5$

5.4

1. 39, 46
2. 15 years, 50 years
3. 22 cm, 11 cm
4. 25

5.5

1. (i) $2(x + y) = 98$

   (ii) $y = 2x + 10$, where age of son = x years, age of father = y years

   (iii) $x + 10 = y$

   (iv) $2x + 3y = 120$

2. True 3. False

5.7

1. $x = 4, y = 1$, unique solution
2. $x = 2, y = -1$, unique solution
3. Infinitely many solutions
4. No solution
5. $x = 2, y = 1$, unique solution

5.8

1. $x = 8, y = 6$ 2. $x = -2, y = 5$
3. $x = 5, y = -2$ 4. $x = 1, y = 3$

5.9

1. $x = 2, y = -3$ 2. $x = 1, y = 2$
3. $x = 11, y = 2$ 4. $x = 1, y = 3$
5. $x = 5, y = -2$ 6. $x = 6, y = -1$

5.10

1. 14 years, 42 years
2. 7 m
3. 75 prizes Rs 500 and 125 prizes of Rs 100 each.
4. 17 of Rs 100 each and 16 of Rs 50 each.

ANSWERS TO TERMINAL EXERCISE

1. (i) (B) (ii) (C) (iii) (A) (iv) (C) (v) (D)
2. (i) $z = 5$ (ii) $x = -8$ (iii) $y = 5$ (iv) $x = 2$
3. 18
4. Age of Reena = 32 years, age of Meena = 40 years
5. $\frac{13}{21}$

6. (i) $x = 1, y = -3$ (ii) $x = 3, y = 4$

   (iii) $x = 3, y = 1$ (iv) $x = 3, y = 5$

7. (i) $x = 1, y = 1$ (ii) $x = 0, y = 1$

   (iii) $x = 3, y = 2$ (iv) $x = -3, y = -4$

8. 74

9. Atul: 19 years, Parul: 7 years

10. 10 m, 6m