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26

INTRODUCTION TO PROBABILITY

In our day to day life, we sometimes make the statements:

(i) It may rain today

(ii) Train is likely to be late

(iii) It is unlikely that bank made a mistake

(iv) Chances are high that the prices of pulses will go down in next september

(v) I doubt that he will win the race.

and so on.

The words **may**, **likely**, **unlikely**, **chances**, **doubt** etc. show that the event, we are talking about , is **not certain to occur**. It may or may not occur. Theory of probability is a branch of mathematics which has been developed to deal with situations involving uncertainty.

The theory had its beginning in the 16th century. It originated in the games of chance such as throwing of dice and now probability is used extensively in biology, economics, genetics, physics, sociology etc.

OBJECTIVES

After studying this lesson, you will be able to

- *understand the meaning of a random experiment;*
- *differentiate between outcomes and events of a random experiment;*
- *define probability P(E) of occurrence of an event E;*
- *determine P($\overline{E}$) if P(E) is given;*
- *state that for the probability P(E), $0 \le P(E) \le 1$;*
- *apply the concept of probability in solving problems based on tossing a coin throwing a die, drawing a card from a well shuffled deck of playing cards, etc.*

EXPECTED BACKGROUND KNOWLEDGE

We assume that the learner is already familiar with

- the term associated with a coin, i.e., head or tail
- a die, face of a die, numbers on the faces of a die
- playing cards - number of cards in a deck, 4- suits of 13 cards-spades, hearts, diamonds and clubs. The cards in each suit such as king, queen, jack etc, are face cards.
- Concept of a ratio/fraction/decimal and operations on them.

26.1 RANDOM EXPERIMENT AND ITS OUTCOMES

Observe the following situations:

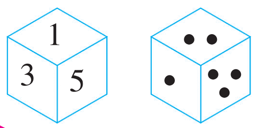
(1) Suppose we toss a coin. We know in advance that the coin can only land in one of two possible ways that is either Head (H) up or Tail (T) up.

(2) Suppose we throw a die. We know in advance that the die can only land in any one of six different ways showing up either 1, 2, 3, 4, 5 or 6.

(3) Suppose we plant 4 seeds and observe the number of seeds germinated after three days. The number of germinated seeds could be either 0, 1, 2, 3, or 4.

When we speak of a coin, we assume it to be fair in the sense that it is symmetrical so that there is no reason for it to land more often on a particular side.

A die is a well balanced cube with its six faces marked with numbers (or dots) from 1 to 6, one number on one face



In the above situations, tossing a coin, throwing a die, planting seeds and observing the germinated seeds, each is an example of a random experiment

In (1), the possible outcomes of the random experiment of tossing a coin are: Head and Tail.

In (2), the possible outcomes of the experiment are: 1, 2, 3, 4, 5, 6

In (3), the possible outcomes are: 0, 1, 2, 3, 4.

A random experiment always has more than one possible outcomes. When the experiment is performed only one outcome out of all possible outcomes comes out. Moreover, we can not predict any particular outcome before the experiment is performed. Repeating the experiment may lead to different outcomes.

Some more examples of random experiments are:

(i) drawing a ball from a bag containing identical balls of different colours without looking into the bag.

(ii) drawing a card at random from a well suffled deck of playing cards

we will now use the word experiment for random experiment throughout this lesson

A deck of playing cards consists of 52 cards which are divided into four suits of 13 cards each-spades (♠) hearts (♥) diamonds (♦) and clubs (♣). Spades and clubs are of black colour and others are of red colour. The cards in each suit are ace, king, queen, jack, 10, 9, 8, 7, 6, 5, 4, 3, and 2. Cards of kings, queens and jacks are called **face cards.**

CHECK YOUR PROGRESS 26.1

1. Which of the following is a random experiment?

   (i) Suppose you guess the answer to a multiple choice question having four options A, B, C, and D, in which only one is correct.

   (ii) The natural numbers 1 to 20 are written on separate slips (one number on one slip) and put in a bag. You draw one slip without looking into the bag.

   (iii) You drop a stone from a height

   (iv) Each of Hari and John chooses one of the numbers 1, 2, 3, independently.

2. What are the possible outcomes of random experiments in Q. 1 above?

26.2 PROBABILITY OF AN EVENT

Suppose a coin is tossed **at random**. We have two possible outcomes, Head (H) and Tail (T). We may assume that each outcome H or T is as likely to occur as the other. In other words, we say that the two outcomes H and T are **equally likely.**

Tossed **at random** means that the coin is allowed to fall freely without any **bias** or interference.

Similarly, when we throw a die, it seems reasonable to assume that each of the six faces (or each of the outcomes 1, 2, 3, 4, 5, 6) is just as likely as any other to occur. In other words, we say that the six outcomes 1, 2, 3, 4, 5 and 6 are **equally likely**.

Before we come to define probability of an event, let us understand the meaning of word **Event**. One or more outcomes constitute an event of an experiment. For example, in throwing a die an event could be "the die shows an even number". This event corresponds to three different outcomes 2, 4 or 6. However, the term event also often used to describe a single outcome. In case of tossing a coin, "the coin shows up a head" or "the coin shows up a tail" each is an event, the first one corresponds to the outcome H and the other to the outcome T. If we write the event E: "the coin shows up a head" If F : " the coin shows up a tail" E and F are called elementary events. An event having only one outcome of the experiment is called an elementary event.

The probability of an event E, written as P(E), is defined as

$$P(E) = \frac{\text{Number of outcomes favourable to E}}{\text{Number of all possible outcomes of the experiment}}$$

assuming the outcomes to be equally likely.

In this lesson, we will take up only those experiments which have equally likely outcomes.

To find probability of some events, let us consider following examples:

Example 26.1: A coin is tossed once. Find the probability of getting (i) a head, (ii) a tail.

Solution: Let E be the event "getting a head"

Possible outcomes of the experiment are : Head (H), Tail (T)

Number of possible outcomes = 2

Number of outcomes favourable to E = 1 (i.e., Head only)

So, probability to E = P(E) = P (getting a head) = P(head)

$$= \frac{\text{Number of outcomes favourable to E}}{\text{Number of all possible outcomes of the experiment}}$$

$$= \frac{1}{2}$$

Similarly, if F is the event "getting a tail", then

$$P(F) = \frac{1}{2}$$

Example 26.2: A die is thrown once. What is the probability of getting a number 3?

Solution: Let E be the event "getting a number 3".

Possible outcomes of the experiment are: 1, 2, 3, 4, 5, 6

Number of possible outcomes = 6

Number of outcomes favourable to E = 1 (i.e., 3)

So, P(E) = P(3) = $\frac{1}{6}$ ← Number of outcomes favourable to E / ← Number of all possible outcomes

Example 26.3: A die is thrown once. Determine the probability of getting a number other than 3?

Solution: Let F be the event "getting a number other than 3" which means "getting a number 1, 2, 4, 5, 6".

Possible outcomes are : 1, 2, 3, 4, 5, 6

Number of possible outcomes = 6

Number of outcomes favourable to F = 5 (i.e., 1, 2, 4, 5, 6)

So, P(F) = $\frac{5}{6}$

Note that event F in Example 26.3 is the same as event 'not E' in Example 26.2.

Example 26.4: A ball is drawn at random from a bag containing 2 red balls, 3 blue balls and 4 black balls. What is the probability of this ball being of (i) red colour (ii) blue colour (iii) black colour (iv) not blue colour?

Solution:

(i) Let E be the event that the drawn ball is of red colour

Number of possible outcomes of the experiment = $\underset{\text{(Red)}}{2} + \underset{\text{(Blue)}}{3} + \underset{\text{(black)}}{4} = 9$

Number of outcomes favourable to E = 2

So, P(Red ball) = P(E) = $\frac{2}{9}$

(ii) Let F be the event that the ball drawn is of blue colour

So, P(Blue ball) = P(F) = $\frac{3}{9} = \frac{1}{3}$

(iii) Let G be the event that the ball drawn is of black colour

So P (Black ball) = P(G) = $\frac{4}{9}$

(iv) Let H be the event that the ball drawn is not of blue colour.

Here "ball of not blue colour" means "ball of red or black colour)

Therefore, number of outcomes favourable to H = 2 + 4 = 6

So, $P(H) = \frac{6}{9} = \frac{2}{3}$

Example 26.5: A card is drawn from a well shuffled deck of 52 playing cards. Find the probability that it is of (i) red colour (ii) black colour

Solution: (i) Let E be the event that the card drawn is of red colour.

Number of cards of red colour = 13 + 13 = 26 (diamonds and hearts)

So, the number of favourable outcomes to E = 26

Total number of cards = 52

Thus, $P(E) = \frac{26}{52} = \frac{1}{2}$

(ii) Let F be the event that the card drawn is of black colour. Number of cards of black colour = 13 + 13 = 26

So $P(F) = \frac{26}{52} = \frac{1}{2}$

Example 26.6: A die is thrown once. What is the probability of getting a number (i) less than 7? (ii) greater than 7?

Solution: (i) Let E be the event "number is less than 7".

Number of favourable outcomes to E = 6 (since every face of a die is marked with a number less than 7)

So, $P(E) = \frac{6}{6} = 1$

(ii) Let F be the event "number is more than 7"

Number of outcomes favourable to F = 0 (since no face of a die is marked with a number more than 7)

So, $P(F) = \frac{0}{6} = 0$

CHECK YOUR PROGRESS 26.2

1. Find the probability of getting a number 5 in a single throw of a die.
2. A die is tossed once. What is the probability that it shows:

   (i) a number 7?

   (ii) a number less than 5?
3. From a pack of 52 cards, a card is drawn at random. What is the probability of this card to be a king?
4. An integer is chosen between 0 and 20. What is the probability that this chosen integer is a prime number?
5. A bag contains 3 red and 3 white balls. A ball is drawn from the bag without looking into it. What is the probability of this ball to be of (i) red colour (ii) white colour?
6. 3 males and 4 females appear for an interview, of which one candidate is to be selected. Find the probability of selection of a (i) male candidate (ii) female candidate.

26.3 MORE ABOUT PROBABILITY

Probability has many interesting properties. We shall explain these through some examples:

Observation 1: In Example 26.6 above,

(a) Event E is sure to occur, since every number on a die is always less than 7. Such an event which is sure to occur is called a sure (or certain) event. Probability of a sure event is taken as 1.

(b) Event F is impossible to occur, since no number on a die is greater than 7. Such an event which is impossible to occur is called an impossible event. Probability of an impossible event is taken as 0.

(c) From the definition of probability of an event E, P(E) cannot be greater than 1, since numerator being the number of outcomes favourable to E cannot be greater than the denominator (number of all possible outcomes).

(d) both the numerator and denominator are natural numbers, so P(E) cannot be negative.

In view of (a), (b), (c) and (d), P(E) takes any value from 0 to 1, i.e.,

$$0 \leq P(E) \leq 1$$

Observation 2: In Example 26.1, both the events getting a head (H) and getting a tail (T) are elementary events and

$$P(H) + P(T) = \frac{1}{2} + \frac{1}{2} = 1$$

Similarly, in the experiment of throwing a die once, elementary events are getting the numbers 1, 2, 3, 4, 5 or 6 and also

$$P(1) + P(2) + P(3) + P(4) + P(5) + P(6) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = 1$$

Observe that the sum of the probabilities of all the elementary events of an experiment is one.

Observation 3: From Examples 26.2 and 26.3,

Probability of getting 3 + Probability of getting a number other than 3 = $\frac{1}{6} + \frac{5}{6} = 1$

i.e. $\quad P(3) + P(\text{not } 3) = 1$

or $\quad P(E) + P(\text{not } E) = 1 \qquad\qquad ...(1)$

Similarly, in Example 26.1

$P(\text{getting a head}) = P(E) = \frac{1}{2}$

$P(\text{getting a tail}) = P(F) = \frac{1}{2}$

So, $P(E) + P(F) = \frac{1}{2} + \frac{1}{2} = 1$

So, $P(E) + P(\text{not } E) = 1$ [getting a tail means getting no head] ...(2)

From (1) and (2), we see that for any event E,

$$P(E) + P(\text{not } E) = 1$$

or $\quad P(E) + P(\overline{E}) = 1 \qquad$ [We denote 'not E' by $\overline{E}$]

Event $\overline{E}$ is called **complement** of the event E or E and $\overline{E}$ are called **complementary events**.

In general, it is true that for an event E

$$P(E) + P(\overline{E}) = 1$$

Example 26.7: If $P(E) = \frac{2}{7}$, what is the probability of 'not E'?

Solution: $P(E) + P(\text{not } E) = 1$

So, $\quad P(\text{not } E) = 1 - P(E) = 1 - \frac{2}{7} = \frac{5}{7}$

Example 26.8: What is the probability that the number 5 will not come up in single throw of a die?

Solution: Let E be the event "number 5 comes up on the die"

Then we have to find P(not E) i.e. $P(\overline{E})$

Now $\qquad P(E) = \frac{1}{6}$

So, $\qquad P(\overline{E}) == 1 - \frac{1}{6} = \frac{5}{6}$

Example 26.9: A card is drawn at random from a well-shuffled deck of 52 cards. Find the probability that this card is a face card.

Solution: Number of all possible outcomes = 52

Number of outcomes favourable to the Event E "a face card" = $3 \times 4 = 12$

[Kings, queens, and jacks are face cards]

So, P(a face card) = $\frac{12}{52} = \frac{3}{13}$

Example 26.10: A coin is tossed two times. What is the probability of getting a head each time?

Solution: Let us write H for Head and T for Tail.

In this expreiment, the possible outcomes will be: HH, HT, TH, TT

HH means Head on both the tosses

HT means Head on 1st toss and Tail on 2nd toss.

TH means Tail on 1st toss and Head on 2nd toss.

TT means Tail on both the tosses.

So, the number of possible outcomes = 4

Let E be the event "getting head each time". This means getting head in both the tosses, i.e. HH.

Therefore, P(HH) = $\frac{1}{4}$

Example 26.11: 10 defective rings are accidentally mixed with 100 good ones in a lot. It is not possible to just look at a ring and tell whether or not it is defective. One ring is drawn at random from this lot. What is the probability of this ring to be a good one?

Solution: Number of all possible outcomes = 10 + 100 = 110

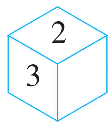
Number of outcomes favourable to the event E " ring is good one" = 100

$$\text{So, } P(E) = \frac{100}{110} = \frac{10}{11}$$

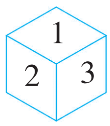
Example 26.12: Two dice, one of black colour and other of blue colour, are thrown at the same time. Write down all the possible outcomes. What is the probability that same number appear on both the dice?

Solution: All the possible outcomes are as given below, where the first number in the bracket is the number appearing on black coloured die and the other number is on blue die.

Blue coloured die



Black coloured die



	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

So, the number of possible outcomes = $6 \times 6 = 36$

The outcomes favourable to the event E : "Same number appears on both dice". are (1, 1), (2, 2), (3, 3), (4, 4), (5, 5) and (6, 6).

So, the number of outcomes favourable to E = 6.

$$\text{Hence, } P(E) = \frac{6}{36} = \frac{1}{6}$$

CHECK YOUR PROGRESS 26.3

1. Complete the following statements by filling in blank spaces:

   (a) The probability of an event is always greater than or equal to ________ but less than or equal to ________

(b) The probability of an event that is certain to occur is ________. Such an event is called ________

(c) The probability of an event which cannot occur is _________. Such an event is __________

(d) The sum of probabilities of two complementary events is _________

(e) The sum of probabilities of all the elementary events of an experiment is ______

2. A die is thrown once. What is the probability of getting

   (a) an even number

   (b) an odd number

   (c) a prime number

3. In Question 2 above, verify:

   P(an even number) + P(an odd number) = 1

4. A die is thrown once. Find the probability of getting

   (i) a number less than 4

   (ii) a number greater than or equal to 4

   (iii) a composite number

   (iv) a number which is not composite

5. If P(E) = 0.88, what is the probability of 'not E'?

6. If $P(\overline{E}) = 0$, find P(E).

7. A card is drawn from a well shuffled deck of 52 playing cards. Find the probability that this card will be

   (i) a red card (ii) a black card

   (iii) a red queen (iv) an ace of black colour

   (v) a jack of spade (vi) a king of club

   (vii) not a face card (viii) not a jack of diamonds

8. A bag contains 15 white balls and 10 blue balls. A ball is drawn at random from the bag. What is the probability of drawing

   (i) a ball of not blue colour (ii) a ball not of white colour

9. In a bag there are 3 red, 4 green and 2 blue marbles. If a marble is picked up at random what is the probability that it is

   (i) not green? (ii) not red? (iii) not blue?

10. Two different coins are tossed at the same time. Write down all possible outcomes. What is the probability of getting head on one and tail on the other coin?

11. In Question 10 above, what is the probability that both the coins show tails?

12. Two dice are thrown simultaneously and the sum of the numbers appearing on them is noted. What is the probability that the sum is

    (i) 7 (ii) 8 (iii) 9 (iv) 10 (v) 12

13. 8 defective toys are accidentally mixed with 92 good ones in a lot of identical toys. One toy is drawn at random from this lot. What is the probability that this toy is defective?

LET US SUM UP

- A random experiment is one which has more than one outcomes and whose outcome is not exactly predictable in advance before performig the experiment.
- One or more outcomes of an experiment constitute an event.
- An event having only one outcome of the experiment is called an elementary event.
- Probability of an event E, P(E), is defined as

  $$P(E) = \frac{\text{Number of outcomes favourable to E}}{\text{Number of all possible outcomes of the experiment}}$$, When the outcomes are equally likely
- $0 \leq P(E) \leq 1$
- If P(E) = 0, E is called an impossible event. If P(E) = 1, E is called a sure or certain event.
- The sum of the probabilities of all the elementary events of an experiment is 1.
- $P(E) + P(\overline{E}) = 1$, where E and $\overline{E}$ are complementary events.

TERMINAL EXERCISE

1. Which of the following statements are True (T) and which are False (F):

   (i) Probability of an event can be 1.01

   (ii) If P(E) = 0.08, then $P(\overline{E}) = 0.02$

(iii) Probability of an impossible event is 1

(iv) For an event E, $0 \le P(E) \le 1$

(v) $P(\overline{E}) = 1 + P(E)$

2. A card is drawn from a well shuffled deck of 52 cards. What is the probability that this card is a face card of red colour?

3. Two coins are tossed at the same time. What is the probability of getting atleast one head? [Hint: P(atleast one head) = 1 – P(no head)]

4. A die is tossed two times and the number appearing on the die is noted each time. What is the probability that the sum of two numbers so obtained is

   (i) greater than 12? (ii) less than 12?

   (iii) greater than 11? (iv) greater than 2?

5. Refer to Question 4 above. What is the probability that the product of two number is 12?

6. Refer to Question 4 above. What is the probability that the difference of two numbers is 2?

7. A bag contains 15 red balls and some green balls. If the probability of drawing a green ball is $\frac{1}{6}$, find the number of green balls.

8. Which of the following can not be the probability of an event?

   (A) $\frac{2}{3}$ (B) – 1.01 (C) 12% (D) 0.3

9. In a single throw of two dice, the probability of getting the sum 2 is

   (A) $\frac{1}{9}$ (B) $\frac{1}{18}$ (C) $\frac{1}{36}$ (D) $\frac{35}{36}$

10. In a simultaneous toss of two coins, the probability of getting one head and one tail is

   (A) $\frac{1}{3}$ (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) $\frac{2}{3}$

ANSWERS TO CHECK YOUR PROGRESS

26.1

1. (i), (ii) and (iii)

2. (i) A, B, C, D (ii) 1, 2, 3, ..., 20

   (iii) (1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)

26.2

1. $\frac{1}{6}$ 2. (i) 0 (ii) $\frac{2}{3}$ 3. $\frac{1}{13}$ 4. $\frac{8}{19}$

5. (i) $\frac{3}{8}$ (ii) $\frac{5}{8}$ 6. (i) $\frac{3}{7}$ (ii) $\frac{4}{7}$

26.3

1. (a) 0, 1 (b) 1, sure or certain event (c) 0, impossible event
   (d) 1 (e) 1

2. (i) $\frac{1}{2}$ (ii) $\frac{1}{2}$ (iii) $\frac{1}{2}$

4. (i) $\frac{1}{2}$ (ii) $\frac{1}{2}$ (iii) $\frac{1}{3}$ (iv) $\frac{2}{3}$

5. 0.12 6. 1

7. (i) $\frac{1}{2}$ (ii) $\frac{1}{2}$ (iii) $\frac{1}{26}$ (iv) $\frac{1}{26}$ (v) $\frac{1}{52}$ (vi) $\frac{1}{52}$
   (vii) $\frac{10}{13}$ (viii) $\frac{51}{52}$

8. (i) $\frac{3}{5}$ (ii) $\frac{2}{5}$

9. (i) $\frac{5}{9}$ (ii) $\frac{2}{3}$ (iii) $\frac{7}{9}$

10. HH, HT, TH, TT, $\frac{1}{2}$

11. $\frac{1}{4}$ 12. (i) $\frac{1}{6}$ (ii) $\frac{5}{36}$ (iii) $\frac{1}{9}$ (iv) $\frac{1}{12}$ (v) $\frac{1}{36}$

13. $\frac{2}{25}$

ANSWERS TO TERMINAL EXERCISE

1. (i) F (ii) T (iii) F (iv) T (v) F

2. $\frac{3}{26}$ 3. $\frac{3}{4}$ 4. (i) 0 (ii) 1 (iii) $\frac{1}{36}$ (iv) 1

5. $\frac{1}{9}$ 6. $\frac{2}{9}$ 7. 3 8. (B) 9. (C) 10. (C)