211en25

25

MEASURES OF CENTRAL TENDENCY

In the previous lesson, we have learnt that the data could be summarised to some extent by presenting it in the form of a frequency table. We have also seen how data were represented graphically through bar graphs, histograms and frequency polygons to get some broad idea about the nature of the data.

Some aspects of the data can be described quantitatively to represent certain features of the data. An average is one of such representative measures. As average is a number of indicating the representative or central value of the data, it lies somewhere in between the two extremes. For this reason, average is called a **measure of central tendency.**

In this lesson, we will study some common measures of central tendency, viz.

(i) Arithmetical average, also called mean

(ii) Median

(iii) Mode

OBJECTIVES

After studying this lesson, you will be able to

- *define mean of raw/ungrouped and grouped data;*
- *calculate mean of raw/ungrouped data and also of grouped data by ordinary and short-cut-methods;*
- *define median and mode of raw/ungrouped data;*
- *calculate median and mode of raw/ungrouped data.*

25.1 ARITHMETIC AVERAGE OR MEAN

You must have heard people talking about average speed, average rainfall, average height, average score (marks) etc. If we are told that average height of students is 150 cm, it does not mean that height of each student is 150 cm. In general, it gives a message that height of students are spread around 150 cm. Some of the students may have a height less than it, some may have a height greater than it and some may have a height of exactly 150 cm.

25.1.1 Mean (Arithmetic average) of Raw Data

To calculate the mean of raw data, all the observations of the data are added and their sum is divided by the number of observations. Thus, the mean of n observations $x_1, x_2, ...x_n$ is

$$\frac{x_1 + x_2 + ... + x_n}{n}$$

It is generally denoted by $\overline{x}$. so

$$\overline{x} = \frac{x_1 + x_2 + ... + x_n}{n}$$

$$= \frac{\sum_{i=1}^{n} x_i}{n} \qquad \text{(I)}$$

where the symbol "Σ" is the capital letter 'SIGMA' of the Greek alphabet and is used to denote summation.

To economise the space required in writing such lengthy expression, we use the symbol Σ, read as **sigma.**

In $\sum_{i=1}^{n} x_i$, i is called the index of summation.

Example 25.1: The weight of four bags of wheat (in kg) are 103, 105, 102, 104. Find the mean weight.

Solution: Mean weight ($\overline{x}$) $= \frac{103+105+102+104}{4}$ kg

$$= \frac{414}{4} \text{ kg} = 103.5 \text{ kg}$$

Example 25.2: The enrolment in a school in last five years was 605, 710, 745, 835 and 910. What was the average enrolment per year?

Solution: Average enrolment (or mean enrolment)

$$= \frac{605+710+745+835+910}{5} = \frac{3805}{5} = 761$$

Example 25.3: The following are the marks in a Mathematics Test of 30 students of Class IX in a school:

40	73	49	83	40	49	27	91	37	31
91	40	31	73	17	49	73	62	40	62
49	50	80	35	40	62	73	49	31	28

Find the mean marks.

Solution: Here, the number of observation (n) = 30

$x_1 = 40,\ x_2 = 73,\ \ldots\ldots\ldots,\ x_{10} = 31$

$x_{11} = 41,\ x_{12} = 40,\ \ldots\ldots\ldots,\ x_{20} = 62$

$x_{21} = 49,\ x_{22} = 50,\ \ldots\ldots\ldots,\ x_{30} = 28$

From the Formula (I), the mean marks of students is given by

$$\text{Mean} = (\bar{x}) = \frac{\sum_{i=1}^{30} x_i}{n} = \frac{40+73+....+28}{30} = \frac{1455}{30}$$

$$= 48.5$$

Example 25.4: Refer to Example 25.1. Show that the sum of $x_1 - \bar{x}$, $x_2 - \bar{x}$, $x_3 - \bar{x}$ and $x_4 - \bar{x}$ is 0, where x_i's are the weights of the four bags and $\bar{x}$ is their mean.

Solution: $x_1 - \bar{x} = 103 - 103.5 = -0.5,\ x_2 - \bar{x} = 105 - 103.5 = 1.5$

$x_3 - \bar{x} = 102 - 103.5 = -1.5,\ x_4 - \bar{x} = 104 - 103.5 = 0.5$

So, $(x_1 - \bar{x}) + (x_2 - \bar{x}) + (x_3 - \bar{x}) + (x_4 - \bar{x}) = -0.5 + 1.5 + (-1.5) + 0.5 = 0$

Example 25.5: The mean of marks obtained by 30 students of Section A of Class X is 48, that of 35 students of Section B is 50. Find the mean marks obtained by 65 students in Class X.

Solution: Mean marks of 30 students of Section A = 48

So, total marks obtained by 30 students of Section A = 30 × 48 = 1440

Similarly, total marks obtained by 35 students of Section B = 35 × 50 = 1750

Total marks obtained by both sections = 1440 + 1750 = 3190

Mean of marks obtained by 65 students = $\frac{3190}{65}$ = 49.1 approx.

Example 25.6: The mean of 6 observations was found to be 40. Later on, it was detected that one observation 82 was misread as 28. Find the correct mean.

Solution: Mean of 6 observations = 40

So, the sum of all the observations = 6 × 40 = 240

Since one observation 82 was misread as 28,

therefore, correct sum of all the observations = 240 – 28 + 82 = 294

Hence, correct mean = $\frac{294}{6}$ = 49

CHECK YOUR PROGRESS 25.1

1. Write formula for calculating mean of n observations $x_1, x_2 ..., x_n$.
2. Find the mean of first ten natural numbers.
3. The daily sale of sugar for 6 days in a certain grocery shop is given below. Calculate the mean daily sale of sugar.

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
74 kg	121 kg	40 kg	82 kg	70.5 kg	130.5 kg

4. The heights of 10 girls were measured in cm and the results were as follows:

   142, 149, 135, 150, 128, 140, 149, 152, 138, 145

   Find the mean height.

5. The maximum daily temperature (in °C) of a city on 12 consecutive days are given below:

32.4	29.5	26.6	25.7	23.5	24.6
24.2	22.4	24.2	23.0	23.2	28.8

   Calcualte the mean daily temperature.

6. Refer to Example 25.2. Verify that the sum of deviations of x_i from their mean ($\bar{x}$) is 0.
7. Mean of 9 observatrions was found to be 35. Later on, it was detected that an observation which was 81, was taken as 18 by mistake. Find the correct mean of the observations.
8. The mean marks obtained by 25 students in a class is 35 and that of 35 students is 25. Find the mean marks obtained by all the students.

25.1.2 Mean of Ungrouped Data

We will explain to find mean of ungrouped data through an example.

Find the mean of the marks (out of 15) obtained by 20 students.

12	10	5	8	15	5	2	8	10	5
10	12	12	2	5	2	8	10	5	10

This data is in the form of raw data. We can find mean of the data by using the formula (I), i.e., $\frac{\sum x_i}{n}$. But this process will be time consuming.

We can also find the mean of this data by first making a frequency table of the data and then applying the formula:

$$\text{mean} = \bar{x} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i} \qquad \text{(II)}$$

where f_i is the frequency of the ith observation x_i.

Frequency table of the data is :

Marks (x_i)	Number of students (f_i)
2	4
5	5
8	3
10	5
12	2
15	1
	$\Sigma f_i = 20$

To find mean of this distribution, we first find $f_i x_i$, by multiplying each x_i with its corresponding frequency f_i and append a column of $f_i x_i$ in the frequency table as given below.

Marks (x_i)	Number of students (f_i)	$f_i x_i$
2	4	$2\times 4 = 8$
5	5	$5\times 5 = 25$
8	3	$3\times 8 = 24$
10	5	$5 \times 10 = 50$
12	2	$2 \times 12 = 24$
15	1	$1 \times 15 = 15$
	$\Sigma f_i = 20$	$\Sigma f_i x_i = 146$

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i} = \frac{146}{20} = 7.3$$

Example 25.7: The following data represents the weekly wages (in rupees) of the employees:

Weekly wages (in ₹)	900	1000	1100	1200	1300	1400	1500
Number of employees	12	13	14	13	14	11	5

Find the mean weekly wages of the employees.

Solution: In the following table, entries in the first column are x_i's and entries in second columen are f_i's, i.e., corresponding frequencies. Recall that to find mean, we require the product of each x_i with corresponding frequency f_i. So, let us put them in a column as shown in the following table:

Weekly wages (in ₹) (x_i)	**Number of employees (f_i)**	$f_i x_i$
900	12	10800
1000	13	13000
1100	14	15400
1200	13	15600
1300	12	15600
1400	11	15400
1500	5	7500
	$\Sigma f_i = 80$	$\Sigma f_i\, x_i = 93300$

Using the Formula II,

$$\text{Mean weekly wages} = \frac{\sum f_i x_i}{\sum f_i} = ₹\,\frac{93300}{80}$$

$$= ₹\ 1166.25$$

Sometimes when the numerical values of x_i and f_i are large, finding the product f_i and x_i becomes tedius and time consuming.

We wish to find a **short-cut method**. Here, we choose an arbitrary constant a, also called the **assumed mean** and subtract it from each of the values x_i. The reduced value, $d_i = x_i - a$ is called the **deviation of x_i from a.**

Thus, $x_i = -\,a + d_i$

and $\quad f_i x_i = af_i + f_i d_i$

$$\sum_{i=1}^{n} f_i x_i = \sum_{i=1}^{n} af_i + \sum_{i=1}^{n} f_i d_i \quad \text{[Summing both sides over } i \text{ from } i \text{ to } r\text{]}$$

Hence $\bar{x} = \sum f_i + \frac{1}{N}\sum f_i d_i$, where $\Sigma f_i = N$

$$\bar{x} = a + \frac{1}{N}\sum f_i d_i \qquad \text{(III)}$$

[since $\Sigma f_i = N$]

This meghod of calcualtion of mean is known as **Assumed Mean Method.**

In Example 25.7, the values x_i were very large. So the product $f_i x_i$ became tedious and time consuming. Let us find mean by **Assumed Mean Method.** Let us take assumed mean $a = 1200$

Weekly wages (in ₹) (x_i)	**Number of employees (f_i)**	**Deviations $d_i = x_i - 1200$**	**$f_i d_i$**
900	12	– 300	– 3600
1000	13	– 200	– 2600
1100	14	– 100	– 1400
1200	13	0	0
1300	12	100	+ 1200
1400	11	200	+ 2200
1500	5	300	+ 1500
	$\Sigma f_i = 80$		$\Sigma f_i d_i = -2700$

Using Formula III,

$$\text{Mean} = a + \frac{1}{N}\sum f_i d_i$$

$$= 1200 + \frac{1}{80}(-2700)$$

$$= 1200 - 33.75 = 1166.25$$

So, the mean weekly wages = ₹ 1166.25

Observe that the mean is the same whether it is calculated by Direct Method or by Assumed Mean Method.

Notes

Example 25.8: If the mean of the following data is 20.2, find the value of k

x_i	10	15	20	25	30
f_i	6	8	20	k	6

Solution: Mean = $\dfrac{\sum f_i x_i}{\sum f_i} = \dfrac{60+120+400+25k+180}{40+k}$

$$= \frac{760+25k}{40+k}$$

So, $\dfrac{760+25k}{40+k} = 20.2$ (Given)

or $760 + 25k = 20.2\ (40 + k)$

or $7600 + 250k = 8080 + 202k$

or $k = 10$

CHECK YOUR PROGRESS 25.2

1. Find the mean marks of the following distribution:

Marks	1	2	3	4	5	6	7	8	9	10
Frequency	1	3	5	9	14	18	16	9	3	2

2. Calcualte the mean for each of the following distributions:

(i)

x	6	10	15	18	22	27	30
f	12	36	54	72	62	42	22

(ii)

x	5	5.4	6.2	7.2	7.6	8.4	9.4
f	3	14	28	23	8	3	1

3. The wieghts (in kg) of 70 workers in a factory are given below. Find the mean weight of a worker.

Weight (in kg)	**Number of workers**
60	10
61	8
62	14
63	16
64	15
65	7

4. If the mean of following data is 17.45 determine the value of p:

x	15	16	17	18	19	20
f	3	8	10	p	5	4

25.1.3 Mean of Grouped Data

Consider the following grouped frequency distribution:

Daily wages (in ₹)	Number of workers
150-160	5
160-170	8
170-180	15
180-190	10
190-200	2

What we can infer from this table is that there are 5 workers earning daily somewhere from ₹ 150 to ₹ 160 (not included 160). We donot know what exactly the earnings of each of these 5 workers are

Therefore, to find mean of the grasped frequency distribution, we make the following assumptions:

Frequency in any class is centred at its class mark or mid point

Now, we can say that there are 5 workers earning a daily wage of ₹ $\frac{150+160}{2}$ = ₹ 155 each, 8 workers earning a daily wage of ₹ $\frac{160+170}{2}$ = ₹ 165, 15 workers aerning a daily wage of ₹ $\frac{170+160}{2}$ = ₹ 175 and so on. Now we can calculate mean of the given data as follows, using the Formula (II)

Daily wages (in ₹)	Number of workers (f_i)	Class marks (x_i)	$f_i x_i$
150-160	5	155	775
160-170	8	165	1320
170-180	15	175	2625
180-190	10	185	850
190-200	2	195	390
	$\Sigma f_i = 40$		$\Sigma f_i x_i = 6960$

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i} = \frac{6960}{40} = 174$$

So, the mean daily wage = ₹ 174

This method of calculate of the mean of grouped data is Direct Method.

We can also find the mean of grouped data by using Formula III, i.e., by **Assumed Mean Method** as follows:

We take assumed mean a = 175

Daily wages (in ₹)	Number of workers (f_i)	Class marks (x_i)	Deviations $d_i = x_i - 175$	$f_i d_i$
150-160	5	155	– 20	– 100
160-170	8	165	– 10	– 80
170-180	15	175	0	0
180-190	10	185	+ 10	100
190-200	2	195	+ 20	40
	$\Sigma f_i = 40$			$\Sigma f_i d_i = -40$

So, using Formula III,

$$\text{Mean} = a + \frac{1}{N}\sum f_i d_i$$

$$= 175 + \frac{1}{40}(-40)$$

$$= 175 - 1 = 174$$

Thus, the mean daily wage = ₹ 174.

Example 25.9: Find the mean for the following frequency distribution by (i) Direct Method, (ii) Assumed Mean Method.

Class	Frequency
20-40	9
40-60	11
60-80	14
80-100	6
100-120	8
120-140	15
140-160	12
Total	75

Solution: (i) **Direct Method**

Class	Frequency (f_i)	Class marks (x_i)	$f_i x_i$
20-40	9	30	270
40-60	11	50	550
60-80	14	70	980
80-100	6	90	540
100-120	8	110	880
120-140	15	130	1950
140-160	12	150	1800
	$\Sigma f_i = 75$		$\Sigma f_i x_i = 6970$

So, mean $= \dfrac{\sum f_i x_i}{\sum f_i} = \dfrac{6970}{75} = 92.93$

(ii) **Assumed mean method**

Let us take assumed mean $= a = 90$

Class	Frequency (f_i)	Class marks (x_i)	Deviation $d_i = x_i - 90$	$f_i d_i$
20-40	9	30	– 60	– 540
40-60	11	50	– 40	– 440
60-80	14	70	– 20	– 280
80-100	6	90	0	0
100-120	8	110	+ 20	160
120-140	15	130	+ 40	600
140-160	12	150	+ 60	720
	$N = \Sigma f_i = 75$			$\Sigma f_i d_i = 220$

$$\text{Mean} = a + \frac{1}{N}\sum f_i d_i = 90 + \frac{220}{75} = 92.93$$

Note that mean comes out to be the same in both the methods.

In the table above, observe that the values in column 4 are all multiples of 20. So, if we divide these value by 20, we would get smaller numbers to multiply with f_i.

Note that, 20 is also the class size of each class.

So, let $u_i = \dfrac{x_i - a}{h}$, where a is the assumed mean and h is the class size.

Now we calculate u_i in this way and then u_if_i and can find mean of the data by using the formula

$$\text{Mean} = \bar{x} = a + \left(\frac{\sum f_i U_i}{\sum f_i}\right) \times h \qquad \text{(IV)}$$

Let us find mean of the data given in Example 25.9

Take $a = 90$. Here $h = 20$

Class	**Frequency (f_i)**	**Class marks (x_i)**	**Deviation $d_i = x_i - 90$**	$u_i = \frac{x_i - a}{h}$	f_iu_i
20-40	9	30	– 60	– 3	– 27
40-60	11	50	– 40	– 2	– 22
60-80	14	70	– 20	– 1	– 14
80-100	6	90	0	0	0
100-120	8	110	+ 20	1	8
120-140	15	130	+ 40	2	30
140-160	12	150	+ 60	3	36
	$\Sigma f_i = 75$				$\Sigma f_iu_i = 11$

Using the Formula (IV),

$$\text{Mean} = \bar{x} = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right) \times h = 90 + \frac{11}{75} \times 20$$

$$= 90 + \frac{220}{75} = 92.93$$

Calculating mean by using Formula (IV) is known as **Step-deviation Method.**

Note that mean comes out to be the same by using Direct Method, Assumed Method or Step Deviation Method.

Example 25.10: Calcualte the mean daily wage from the following distribution by using Step deviation method.

Daily wages (in ₹)	150-160	160-70	170-180	180-190	190-200
Numbr of workers	5	8	15	10	2

Solution: We have already calculated the mean by using Direct Method and Assumed Method. Let us find mean by Step deviation Method.

Let us take $a = 175$. Here $h = 10$

Daily wages (in ₹)	Number of workers (f_i)	Class marks (x_i)	Deviation $d_i = x_i - 90$	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
150-160	5	155	– 20	– 2	– 10
160-170	8	165	– 10	– 1	– 8
170-180	15	175	0	0	0
180-190	10	185	10	1	10
190-200	2	195	20	2	4
	$\Sigma f_i = 40$				$\Sigma f_i u_i = -4$

Using Formula (IV),

$$\text{Mean daily wages} = a + \left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right) \times h = 175 + \frac{-4}{40} \times 10 = ₹\,174$$

Note: Here again note that the mean is the same whether it is calculated using the Direct Method, Assumed mean Method or Step deviation Method.

CHECK YOUR PROGRESS 25.3

1. Following table shows marks obtained by 100 students in a mathematics test

Marks	0-10	10-20	20-30	30-40	40-50	50-60
Number of students	12	15	25	25	17	6

Calculate mean marks of the students by using Direct Method.

2. The following is the distribution of bulbs kept in boxes:

Number of bulbs	50-52	52-54	54-56	56-58	58-60
Number of boxes	15	100	126	105	30

Find the mean number of bulbs kept in a box. Which method of finding the mean did you choose?

3. The weekly observations on cost of living index in a certain city for a particular year are given below:

Cost of living index	140-150	150-160	160-170	170-180	180-190	190-200
Number of weeks	5	8	20	9	6	4

Calculate mean weekly cost of living index by using Step deviation Method.

4. Find the mean of the following data by using (i) Assumed Mean Method and (ii) Step deviation Method.

Class	150-200	200-250	250-300	300-350	350-400
Frequency	48	32	35	20	10

25.2 MEDIAN

In an office there are 5 employees: a superviosor and 4 workers. The workers draw a salary of ₹ 5000, ₹ 6500, ₹ 7500 and ₹ 8000 per month while the supervisor gets ₹ 20000 per month.

In this case mean (salary) $= ₹\,\dfrac{5000 + 6500 + 7500 + 8000 + 20000}{5}$

$$= ₹\,\frac{47000}{5} = ₹\ 9400$$

Note that 4 out of 5 employees have their salaries much less than ₹ 9400. The mean salary ₹ 9400 does not given even an approximate estimate of any one of their salaries.

This is a weakness of the mean. It is affected by the **extreme** values of the observations in the data.

This weekness of mean drives us to look for another average which is unaffected by a few extreme values. Median is one such a measure of central tendency.

Median is a measure of central tendency which gives the value of the middle-most observation in the data when the data is arranged in ascending (or descending) order.

25.2.1 Median of Raw Data

Median of raw data is calculated as follows:

(i) Arrange the (numerical) data in an ascending (or descending) order

(ii) When the number of observations (n) is **odd**, the median is the value of $\left(\dfrac{n+1}{2}\right)$th observation.

(iii) When the number of observations (n) is **even**, the median is the mean of the $\left(\frac{n}{2}\right)$th and $\left(\frac{n}{2}+1\right)$th observations.

Let us illustrate this with the help of some examples.

Example 25.11: The weights (in kg) of 15 dogs are as follows:

9, 26, 10 , 22, 36, 13, 20, 20, 10, 21, 25, 16, 12, 14, 19

Find the median weight.

Solution: Let us arrange the data in the **ascending (or descending)** order:

9, 10, 10, 12, 13, 14, 16, 19, 20, 20, 21, 22, 25, 36

Median

Here, number of observations = 15

So, the median will be $\left(\frac{n+1}{2}\right)$th, i.e., $\left(\frac{15+1}{2}\right)$th, i.e., 8th observation which is 19 kg.

Remark: The median weight 19 kg conveys the information that 50% dogs have weights less than 19 kg and another 50% have weights more then 19 kg.

Example 25.12: The points scored by a basket ball team in a series of matches are as follows:

16, 1, 6, 26, 14, 4, 13, 8, 9, 23, 47, 9, 7, 8, 17, 28

Find the median of the data.

Solution: Here number of observations = 16

So, the median will be the mean of $\left(\frac{16}{2}\right)$th and $\left(\frac{16}{2}+1\right)$th, i.e., mean of 6th and 9th observations, when the data is arranged in ascending (or descending) order as:

1, 4, 6, 7, 8, 8, 9, 9, 13, 14, 16, 17, 23, 26, 28, 47

8th term 9th term

So, the median $= \frac{9+13}{2} = 11$

Remark: Here again the median 11 conveys the information that the values of 50% of the observations are less than 11 and the values of 50% of the observations are more than 11.

25.2.2 Median of Ungrouped Data

We illustrate caluculation of the median of ungrouped data through examples.

Example 25.13: Find the median of the following data, which gives the marks, out of 15, obtaine by 35 students in a mathematics test.

Marks obtained	3	5	6	11	15	14	13	7	12	10
Number of Students	4	6	5	7	1	3	2	3	3	1

Solution: First arrange marks in ascending order and prepare a frequency table as follows:

Marks obtained	3	5	6	7	10	11	12	13	14	15
Number of Students (frequency)	4	6	5	3	1	7	3	2	3	1

Here $n = 35$, which is odd. So, the median will be $\left(\frac{n+1}{2}\right)$th, i.e., $\left(\frac{35+1}{2}\right)$th, i.e., 18th observation.

To find value of 18th observation, we prepare cumulative frequency table as follows:

Marks obtained	Number of students	Cumulative frequency
3	4	4
5	6	10
6	5	15
7	3	18
10	1	19
11	7	26
12	3	29
13	2	31
14	3	34
15	1	35

From the table above, we see that 18th observation is 7

So, Median = 7

Example 25.14: Find the median of the following data:

Weight (in kg)	40	41	42	43	44	45	46	48
Number of students	2	5	7	8	13	26	6	3

Solution: Here $n = 2 + 5 + 7 + 8 + 13 + 26 + 6 + 3 = 70$, which is even, and weight are already arranged in the ascending order. Let us prepare cumulative frequency table of the data:

Weight (in kg)	Number of students (frequency)	Cumulative frequency	
40	2	2	
41	5	7	
42	7	14	
43	8	22	
44	13	35	35th observation
45	26	61	36th observation
46	6	67	
48	3	70	

Since n is even, so the median will be the mean of $\left(\frac{n}{2}\right)$th and $\left(\frac{n}{2}+1\right)$th observations, i.e., 35th and 36th observations. From the table, we see that

35 the observation is 44

and 36th observation is 45

So, Median $= \frac{44+45}{2} = 44.5$

CHECK YOUR PROGRESS 25.4

1. Following are the goals scored by a team in a series of 11 matches

   1, 0 , 3, 2, 4, 5, 2, 4, 4, 2, 5

   Determine the median score.

2. In a diagnostic test in mathematics given to 12 students, the following marks (out of 100) are recorded

   46, 52, 48, 39, 41, 62, 55, 53, 96, 39, 45, 99

   Calculate the median for this data.

3. A fair die is thrown 100 times and its outcomes are recorded as shown below:

Outcome	1	2	3	4	5	6
Frequency	17	15	16	18	16	18

Find the median outcome of the distributions.

4. For each of the following frequency distributions, find the median:

(a)

x_i	2	3	4	5	6	7
f_i	4	9	16	14	11	6

(b)

x_i	5	10	15	20	25	30	35	40
f_i	3	7	12	20	28	31	28	26

(c)

x_i	2.3	3	5.1	5.8	7.4	6.7	4.3
f_i	5	8	14	21	13	5	7

25.3 MODE

Look at the following example:

A company produces readymade shirts of different sizes. The company kept record of its sale for one week which is given below:

size (in cm)	90	95	100	105	110	115
Number of shirts	50	125	190	385	270	28

From the table, we see that the sales of shirts of size 105 cm is maximum. So, the company will go ahead producing this size in the largest number. Here, 105 is nothing but the **mode** of the data. Mode is also **one of the measures of central tendency.**

The observation that occurs most frequently in the data is called mode of the data.

In other words, the observation with maximum frequency is called mode of the data.

The readymade garments and shoe industries etc, make use of this measure of central tendency. Based on mode of the demand data, these industries decide which size of the product should be produced in large numbers to meet the market demand.

25.3.1 Mode of Raw Data

In case of raw data, it is easy to pick up mode by just looking at the data. Let us consider the following example:

Example 25.15: The number of goals scored by a football team in 12 matches are:

1, 2, 2, 3, 1, 2, 2, 4, 5, 3, 3, 4

What is the modal score?

Solution: Just by looking at the data, we find the frequency of 2 is 4 and is more than the frequency of all other scores.

So, mode of the data is 2, or modal score is 2.

Example 25.16: Find the mode of the data:

9, 6, 8, 9, 10, 7, 12, 15, 22, 15

Solution: Arranging the data in increasing order, we have

6, 7, 8, 9, 9, 10, 12, 15, 15, 22

We find that the both the observations 9 and 15 have the same maximum frequency 2. So, both are the modes of the data.

Remarks: 1. In this lesson, we will take up the data having a single mode only.

2. In the data, if each observation has the same frequency, then we say that the data does not have a mode.

25.3.2 Mode of Ungrouped Data

Let us illustrate finding of the mode of ungrouped data through an example

Example 25.17: Find the mode of the following data:

Weight (in kg)	40	41	42	43	44	45	46	48
Number of Students	2	6	8	9	10	22	13	5

Solution: From the table, we see that the weight 45 kg has maximum frequency 22 which means that maximum number of students have their weight 45 kg. So, the mode is 45 kg or the modal weight is 45 kg.

CHECK YOUR PROGRESS 25.5

1. Find the mode of the data:

   5, 10, 3, 7, 2, 9, 6, 2, 11, 2

2. The number of TV sets in each of 15 households are found as given below:

   2, 2, 4, 2, 1, 1, 1, 2, 1, 1, 3, 3, 1, 3, 0

   What is the mode of this data?

3. A die is thrown 100 times, giving the following results

Outcome	1	2	3	4	5	6
Frequency	15	16	16	15	17	20

Find the modal outcome from this distribution.

4. Following are the marks (out of 10) obtained by 80 students in a mathematics test:

Marks obtained	0	1	2	3	4	5	6	7	8	9	10
Number of students	5	2	3	5	9	11	15	16	9	3	2

Determine the modal marks.

LET US SUM UP

- Mean, median and mode are the measures of central tendency.

- Mean (Arithmetic average) of raw data is givne by $\overline{x} = \dfrac{\sum_{i=1}^{n} x_i}{n}$

  where $x_1, x_2 \ldots., x_n$ are n observations.

- Mean of ungrouped data is given by $\overline{x} = \dfrac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i} = \dfrac{\sum f_i x_i}{N}$

  where f_i is the frequency of the ith observation x_i.

- Mean of ungrouped data can also be found by using the formula $\overline{x} = a + \dfrac{1}{N}\sum f_i d_i$

  where $d_i = x_i - a$, a being the assumed mean

Mean of grouped data

(i) To find mean of the grouped frequency distribution, we take the assumption:

Frequency in any class is centred at its class mark or mid point.

(ii) **Driect Method**

$$\bar{x} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$$

where x_i's are the class marks and f_i are the corresponding freqeucies of x_i's.

(iii) **Assumed Mean Method**

$$\bar{x} = a + \frac{\sum_{i=1}^{n} f_i d_i}{\mathrm{N}}$$

where a is the assumed mean, and $d_i = x_i - a$.

(iv) **Step deviation method**

$$\bar{x} = a + \left(\frac{\sum_{i=1}^{n} f_i u_i}{\sum_{i=1}^{n} f_i} \right) \times h$$

where a is the assumed mean, $u_i = \dfrac{x_i - a}{h}$ and h is the class size.

- Median is a measure of central tendency which gives the value of the middle most obseration in the data, when the data is arranged in ascending (or descending) order.
- **Median of raw data**

  (i) When the number of observations (n) is odd, the median is the value of $\left(\dfrac{n+1}{2}\right)$th observation.

  (ii) When the number of observations (n) is even, the median is the mean of the $\left(\dfrac{n}{2}\right)$th and $\left(\dfrac{n}{2}+1\right)$th observations.

- **Median of ungrouped data**

  Median of ungrouped data can be found from the cumulative frequency table (arranging data in increasing or decreasing order) using (i) and (ii) above.

- The value of observation with maximum frequency is called the mode of the data.

TERMINAL EXERCISE

1. Find the mean of first five prime numbers.
2. If the mean of 5, 7, 9, x, 11 and 12 is 9, find the value of x.
3. Following are the marks obtained by 9 students in a class

   51, 36, 63, 46, 38, 43, 52, 42 and 43

   (i) Find the mean marks of the students.

   (ii) What will be the mean marks if a student scoring 75 marks is also included in the class.
4. The mean marks of 10 students in a class is 70. The students are divided into two groups of 6 and 4 respectively. If the mean marks of the first group is 60, what will be the mean marks of the second group?
5. If the mean of the observations $x_1, x_2, ..., x_n$ is $\bar{x}$, show that $\sum_{i=1}^{n} (x_1 - \bar{x}) = 0$
6. There are 50 numbers. Each number is subtracted from 53 and the mean of the numbers so obtained is found to be – 3.5. Determine the mean of the given numbers.
7. Find the mean of the following data:

(a)

x_i	5	9	13	17	22	25
f_i	3	5	12	8	7	5

(b)

x_i	16	18	28	22	24	26
f_i	1	3	5	7	5	4

8. Find the mean of the following data

(a)

Classes	10-20	20-30	30-40	40-50	50-60	60-70
Frequencies	2	3	5	7	5	3

(b)

Classes	100-200	200-300	300-400	400-500	500-600	600-700
Frequencies	3	5	8	6	5	3

(c) The ages (in months) of a group of 50 students are as follows. Find the mean age.

Age	156-158	158-160	160-162	162-164	164-166	166-168
Number of students	2	4	8	16	14	6

9. Find the median of the following data:

   (a) 5, 12, 16, 18, 20, 25, 10

   (b) 6, 12, 9, 10, 16, 28, 25, 13, 15, 17

   (c) 15, 13, 8, 22, 29, 12, 14, 17, 6

10. The following data are arranged in ascending order and the median of the data is 60. Find the value of x.

    26, 29, 42, 53, x, $x + 2$, 70, 75, 82, 93

11. Find the median of the following data:

(a)

x_i	25	30	35	45	50	55	65	70	85
f_i	5	14	12	21	11	13	14	7	3

(b)

x_i	35	36	37	38	39	40	41	42
f_i	2	3	5	4	7	6	4	2

12. Find the mode of the following data:

    (a) 8, 5, 2, 5, 3, 5, 3, 1

    (b) 19, 18, 17, 16, 17, 15, 14, 15, 17, 9

13. Find the mode of the following data which gives life time (in hours) of 80 bulbs selected at random from a lot.

Life time (in hours)	300	500	700	900	1100
Number of bulbs	10	12	20	27	11

14. In the mean of the following data is 7, find the value of p:

x_i	4	p	6	7	9	11
f_i	2	4	6	10	6	2

15. For a selected group of people, an insurance company recorded the following data:

Age (in years)	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Number of deaths	2	12	55	95	71	42	16	7

    Determine the mean of the data.

16. If the mean of the observations: $x + 1$, $x + 4$, $x + 5$, $x + 8$, $x + 11$ is 10, the mean of the last three observations is

    (A) 12.5 (B) 12.2 (C) 13.5 (D) 14.2

17. If each observation in the data is increased by 2, than their mean

    (A) remains the same (B) becomes 2 times the original mean

    (C) is decreased by 2 (D) is increased by 2

18. Mode of the data: 15, 14, 19, 20, 14, 15, 14, 18, 14, 15, 17, 14, 18 is

    (A) 20 (B) 18 (C) 15 (D) 14

ANSWERS TO CHECK YOUR PROGRESS

25.1

1. $\sum_{i=1}^{n} x_i / n$ 2. 5.5 3. 86.33 kg

4. 142.8 cm 5. 25.68°C 7. 42

8. 29.17

25.2

1. 5.84 2. (i) 18.99 (ii) 6.57

3. 11.68 4. 10

25.3

1. 28.80 2. 55.19 3. 167.9 4. 244.66

25.4

1. 3 2. 50 3. 4

4. (a) 4 (b) 30 (c) 5.8

25.5

1. 2 2. 1 3. 6 4. 7

ANSWERS TO TERMINAL EXERCISE

1. 5.6 2. 10 3. (i) 46 (ii) 48.9

4. 85 6. 56.5 7. (a) 15.775 (b) 21.75

8. (a) 42.6 (b) 396.67 (c) 163 months (approx)

9. (a) 16 (b) 14 (c) 14

10. 59 11. (a) 45 (b) 24 12. (a) 5 (b) 17

13. 900 14. 5 15. 39.86 years 16. (A)

17. (D) 18. D