13

211en13

QUADRILATERALS

If you look around, you will find many objects bounded by four line-segments. Any surface of a book, window door, some parts of window-grill, slice of bread, the floor of your room are all examples of a closed figure bounded by four line-segments. Such a figure is called a quadrilateral.

The word quadrilateral has its origin from the two words "quadric" meaning four and "lateral" meaning sides. Thus, a quadrilateral is that geometrical figure which has four sides, enclosing a part of the plane.

In this lesson, we shall study about terms and concepts related to quadrilateral with their properties.

OBJECTIVES

After studying this lesson, you will be able to

- *describe various types of quadrilaterals viz. trapeziums, parallelograms, rectangles, rhombuses and squares;*
- *verify properties of different types of quadrilaterals;*
- *verify that in a triangle the line segment joining the mid-points of any two sides is parallel to the third side and is half of it;*
- *verify that the line drawn through the mid-point of a side of a triangle parallel to another side bisects the third side;*
- *verify that if there are three or more parallel lines and the intercepts made by them on a transversal are equal, the corresponding intercepts on any other transversal are also equal;*
- *verify that a diagonal of a parallelogram divides it into two triangles of equal area;*
- *solve problem based on starred results and direct numerical problems based on unstarred results given in the curriculum;*

- *prove that parallelograms on the same or equal bases and between the same parallels are equal in area;*
- *verify that triangles on the same or equal bases and between the same parallels are equal in area and its converse.*

EXPECTED BACKGROUND KNOWLEDGE

- Drawing line-segments and angles of given measure.
- Drawing circles/arcs of given radius.
- Drawing parallel and perpendicular lines.
- Four fundamental operations on numbers.

13.1 QUADRILATERAL

Recall that if A, B, C and D are four points in a plane such that no three of them are collinear and the line segments AB, BC, CD and DA do not intersect except at their end points, then the closed figure made up of these four line segments is called a quadrilateral with vertices A, B, C and D. A quadrilateral with vertices A, B, C and D is generally denoted by quad. ABCD. In Fig. 13.1 (i) and (ii), both the quadrilaterals can be named as quad. ABCD or simply ABCD.

In quadrilateral ABCD,

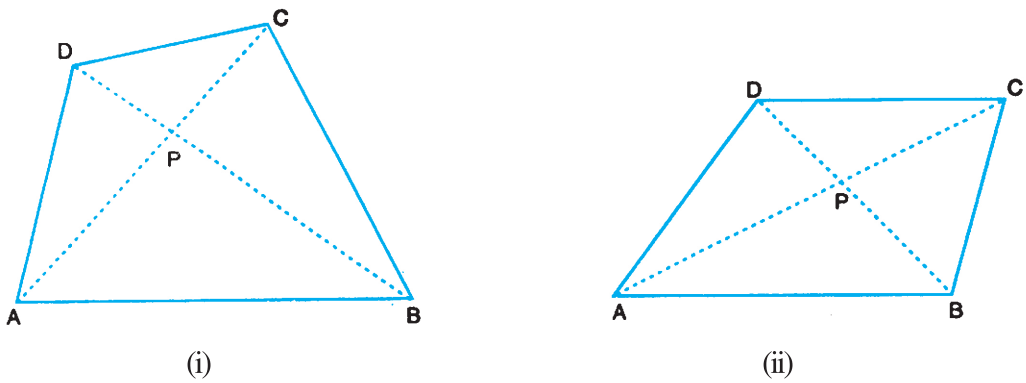

Fig. 13.1

(i) AB and DC ; BC and AD are two pairs of opposite sides.

(ii) $\angle A$ and $\angle C$; $\angle B$ and $\angle D$ are two pairs of opposite angles.

(iii) AB and BC ; BC and CD are two pairs of consecutive or adjacent sides. Can you name the other pairs of consecutive sides?

(iv) $\angle A$ and $\angle B$; $\angle B$ and $\angle C$ are two pairs of consecutive or adjacent angles. Can you name the other pairs of consecutive angles?

(v) AC and BD are the two diagonals.

In Fig. 13.2, angles denoted by l, 2, 3 and 4 are the interior angles or the angles of the quad. ABCD. Angles denoted by 5, 6, 7 and 8 are the exterior angles of the quad. ABCD.

Measure $\angle 1$, $\angle 2$, $\angle 3$ and $\angle 4$.

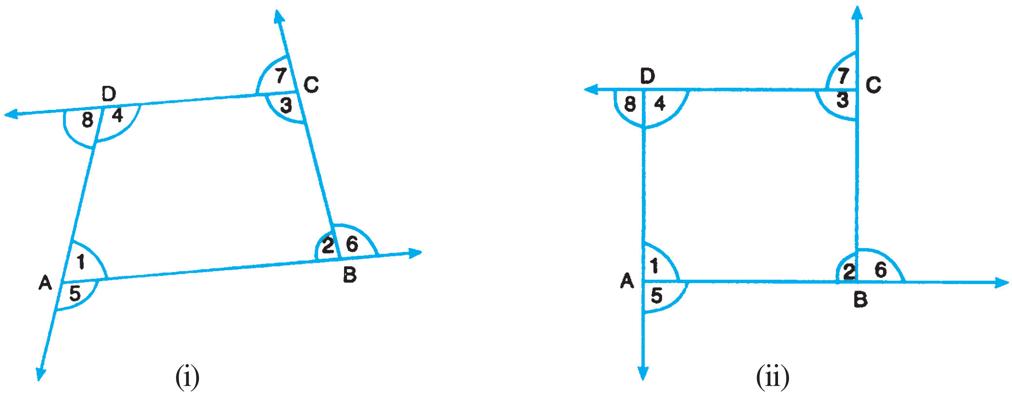

Fig. 13.2

What *is* the sum of these angles You will find that $\angle 1 + \angle 2 + \angle 3 + \angle 4 = 360°$.

i.e. sum of interior angles of a quadrilateral equals 360°.

Also what is the sum of exterior angles of the quadrilateral ABCD?

You will again find that $\angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$

i.e., sum of exterior angles of a quadrilateral is also 360°.

13.2 TYPES OF QUADRILATERALS

You are familiar with quadrilaterals and their different shapes. You also know how to name them. However, we will now study different types of quadrilaterals in a systematic way. A family tree of quadrilaterals is given in Fig. 13.3 below:

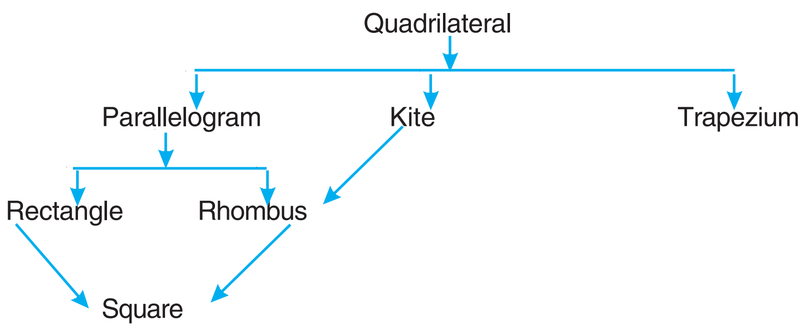

Fig. 13.3

Let us describe them one by one.

1. Trapezium

A quadrilateral which has only one pair of opposite sides parallel is called a trapezium. In

Fig. 13.4 [(i) and (ii)] ABCD and PQRS are trapeziums with AB || DC and PQ || SR respectively.

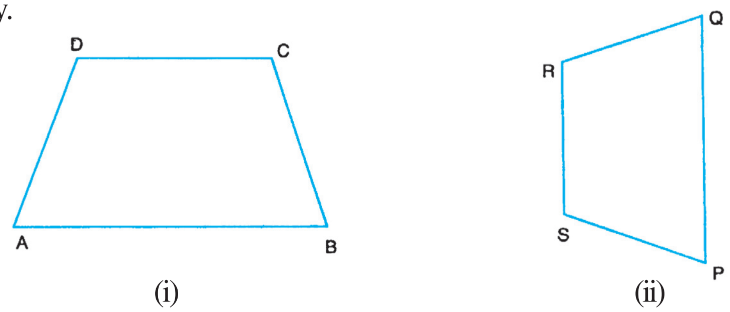

Fig. 13.4

2. Kite

A quadrilateral, which has two pairs of equal sides next to each other, is called a kite. Fig. 13.5 [(i) and (ii)] ABCD and PQRS are kites with adjacent sides AB and AD, BC and CD in (i) PQ and PS, QR and RS in (ii) being equal.

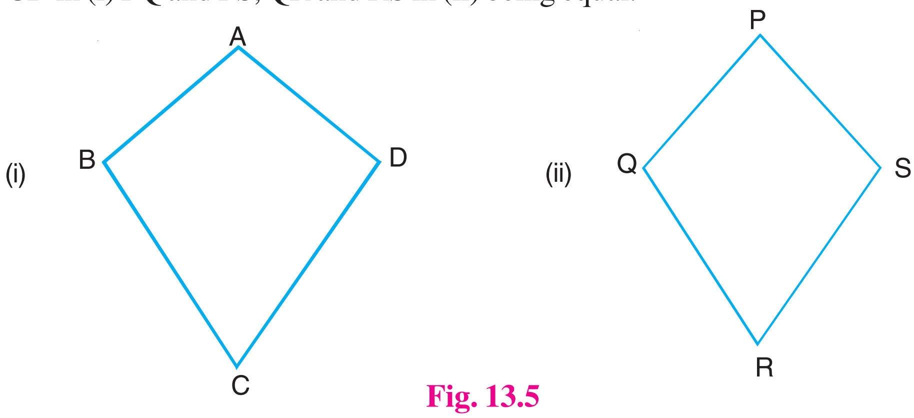

Fig. 13.5

3. Parallelogram

A quadrilateral which has both pairs of opposite sides parallel, is called a parallelogram. In Fig. 13.6 [(i) and (ii)] ABCD and PQRS are parallelograms with AB||DC, AD||BC and PQ||SR, SP||RQ. These are denoted by $||^{gm}$ ABCD (Parallelogram ABCD) and $||^{gm}$ PQRS (Parallelogram PQRS).

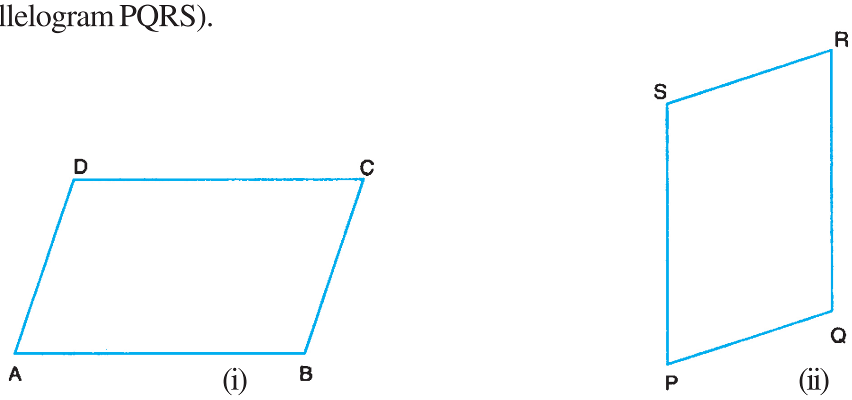

Fig. 13.6

4. Rhombus

A rhombus is a parallelogram in which any pair of adjacent sides is equal.

In Fig. 13.7 ABCD is a rhombus.

You may note that ABCD is a parallelogram with AB = BC = CD = DA i.e., each pair of adjacent sides being equal.

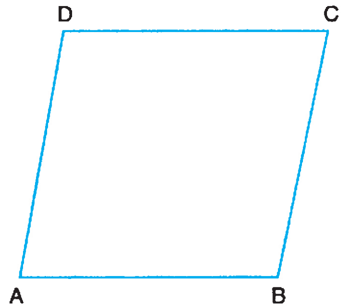

Fig. 13.7

5. Rectangle

A parallelogram one of whose angles is a right angle is called a rectangle.

In Fig. 13.8, ABCD is a rectangle in which AB||DC, AD||BC

and $\angle A = \angle B = \angle C = \angle D = 90^o$.

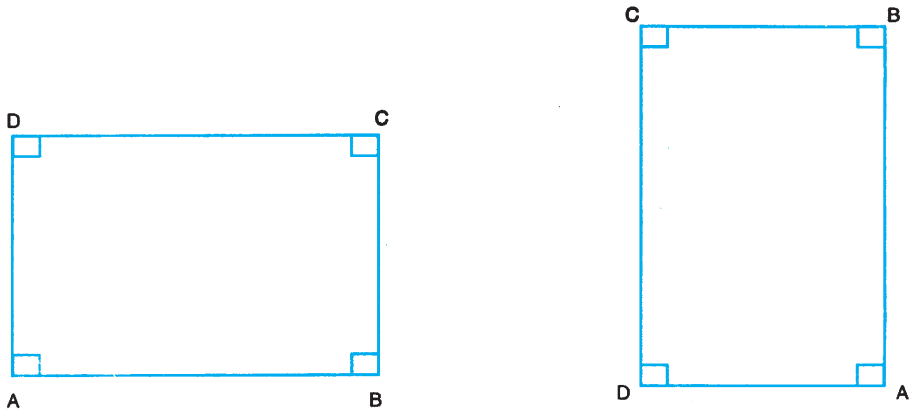

Fig. 13.8

6. Square

A square is a rectangle, with a pair of adjacent sides equal.

In other words, a parallelogram having all sides equal and each angle a right angle is called a square.

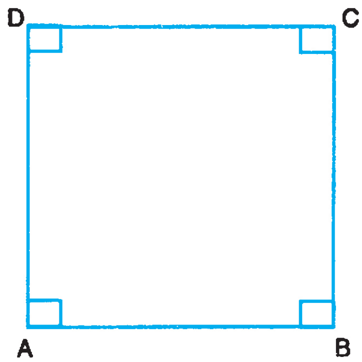

Fig. 13.9

In Fig. 13.9, ABCD is a square in which AB||DC, AD||BC, and AB = BC = CD = DA and $\angle A = \angle B = \angle C = \angle D = 90^\circ$.

Let us take some examples to illustrate different types of quadrilaterals.

Example 13.1: In Fig 13.10, PQR is a triangle. S and T are two points on the sides PQ and PR respectively such that ST||QR. Name the type of quadrilateral STRQ so formed.

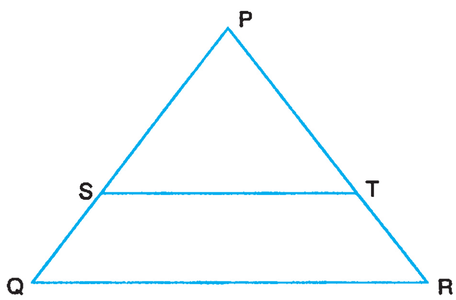

Fig. 13.10

Solution: Quadrilateral STRQ is a trapezium, because ST||QR.

Example 13.2: The three angles of a quadrilateral are 100°, 50° and 70°. Find the measure of the fourth angle.

Solution: We know that the sum of the angles of a quadrilateral is 360°.

Then $\quad 100^\circ + 50^\circ + 70^\circ + x^\circ = 360^\circ$

$$220^\circ + x^\circ = 360^\circ$$

$$x = 140$$

Hence, the measure of fourth angle is 140°.

CHECK YOUR PROGRESS 13.1

1. Name each of the following quadrilaterals.

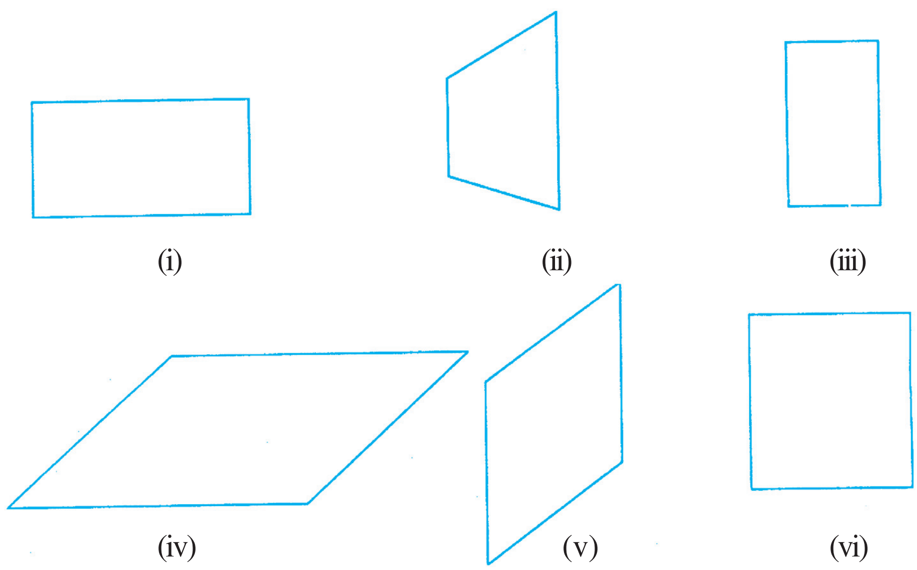

Fig. 13.10

2. State which of the following statements are correct ?

   (i) Sum of interior angles of a quadrilateral is 360°.

   (ii) All rectangles are squares,

   (iii) A rectangle is a parallelogram.

   (iv) A square is a rhombus.

   (v) A rhombus is a parallelogram.

   (vi) A square is a parallelogram.

   (vii) A parallelogram is a rhombus.

   (viii) A trapezium is a parallelogram.

   (ix) A trapezium is a rectangle.

   (x) A parallelogram is a trapezium.

3. In a quadrilateral, all its angles are equal. Find the measure of each angle.

4. The angles of a quadrilateral are in the ratio 5:7:7: 11. Find the measure of each angle.

5. If a pair of opposite angles of a quadrilateral are supplementary, what can you say about the other pair of angles?

13.3 PROPERTIES OF DIFFERENT TYPES OF QUADRILATERALS

1. Properties of a Parallelogram

We have learnt that a parallelogram is a quadrilateral with both pairs of opposite sides parallel. Now let us establish some relationship between sides, angles and diagonals of a parallelogram.

Draw a pair of parallel lines l and m as shown in Fig. 13.12. Draw another pair of parallel lines p and q such that they intersect l and m. You observe that a parallelogram ABCD is formed. Join AC and BD. They intersect each other at O.

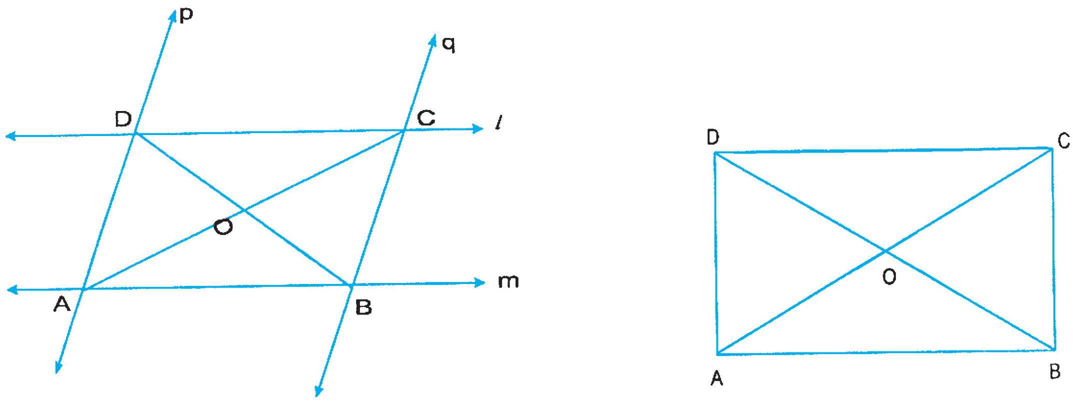

Fig. 13.12

Now measure the sides AB, BC, CD and DA. What do you find?

You will find that AB = DC and BC = AD.

Also measure $\angle ABC$, $\angle BCD$, $\angle CDA$ and $\angle DAB$.

What do you find?

You will find that $\angle DAB = \angle BCD$ and $\angle ABC = \angle CDA$

Again, Measure OA, OC, OB and OD.

What do you find?

You will find that OA = OC and OB = OD

Draw another parallelogram and repeat the activity. You will find that

The opposite sides of a parallelogram are equal.

The opposite angles of a parallelogram are equal.

The diagonals of a parallelogram bisect each other.

The above mentioned properties of a parallelogram can also be verified by Cardboard model which is as follows:

Let us take a cardboard. Draw any parallelogram ABCD on it. Draw its diagonal AC as shown in Fig 13.13 Cut the parallelogram ABCD from the cardboard. Now cut this parallelogram along the diagonal AC. Thus, the parallelogram has been divided into two parts and each part is a triangle.

In other words, you get two triangles, ΔABC and ΔADC. Now place ΔADC on ΔABC in such a way that the vertex D falls on the vertex B and the side CD falls along the side AB.

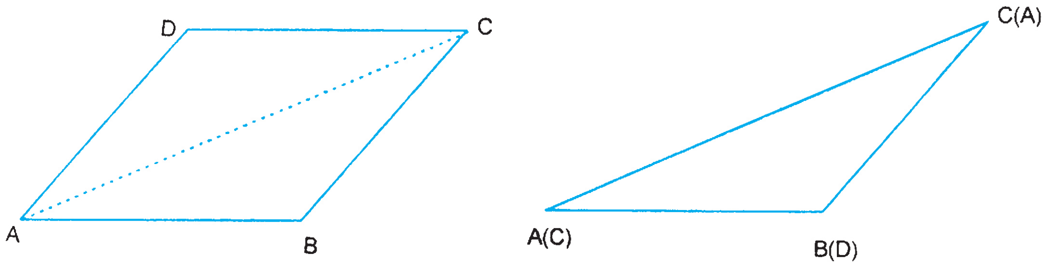

Fig. 13.13

Where does the point C fall?

Where does the point A fall?

You will observe that ΔADC will coincide with ΔABC. In other words $\Delta ABC \cong \Delta ADC$. Also $AB = CD$ and $BC = AD$ and $\angle B = \angle D$.

You may repeat this activity by taking some other parallelograms, you will always get the same results as verified earlier, thus, proving the above two properties of the parallelogram.

Now you can prove the third property of the parallelogram, i.e., the diagonals of a parallelogram bisect each other.

Again take a thin cardboard. Draw any parallelogram PQRS on it. Draw its diagonals

PR and QS which intersect each other at O as shown in Fig. 13.14. Now cut the parallelogram PQRS.

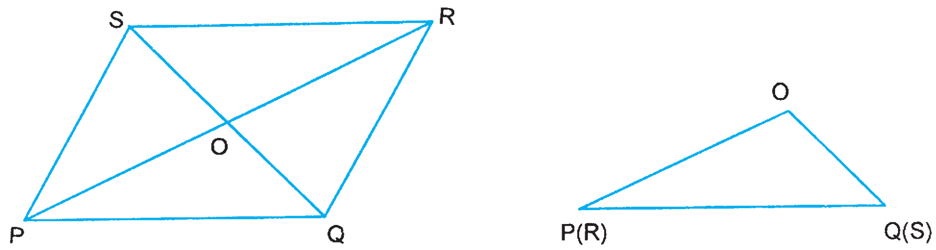

Fig. 13.14

Also cut ΔPOQ and ΔROS.

Now place ΔROS and ΔPOQ in such a way that the vertex R coincides with the vertex P and RO coincides with the side PO.

Where does the point S fall?

Where does the side OS fall?

Is $\Delta ROS \cong \Delta POQ$? Yes, it is.

So, what do you observe?

We find that RO = PO and OS = OQ

You may also verify this property by taking another pair of triangles i.e. ΔPOS and ΔROQ You will again arrive at the same result.

You may also verify the following properties which are the converse of the properties of a parallelogram verified earlier.

A quadrilateral is a parallelogram if its opposite sides are equal.

A quadrilateral is a parallelogram if its opposite angles are equal.

A quadrilateral is a parallelogram if its diagonals bisect each other.

2. Properties of a Rhombus

In the previous section we have defined a rhombus. We know that a rhombus is a parallelogram in which a pair of adjacent sides is equal. In Fig. 13.15, ABCD is a rhombus.

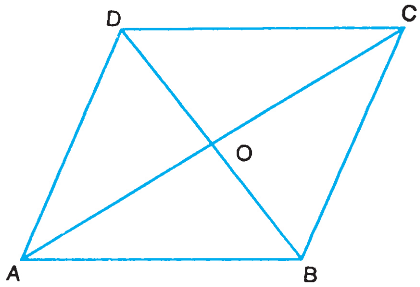

Fig. 13.15

Thus, ABCD is a parallelogram with AB = BC. Since every rhombus is a parallelogram, therefore all the properties of a parallelogram are also true for rhombus, i.e.

(i) Opposite sides are equal,

i.e., AB = DC and AD = BC

(ii) Opposite angles are equal,

i.e., $\angle$ A = $\angle$ C and $\angle$ B = $\angle$ D

(iii) Diagonals bisect each other

i.e., AO = OC and DO = OB

Since adjacent sides of a rhombus are equal and by the property of a parallelogram opposite sides are equal. Therefore,

AB = BC = CD = DA

Thus, all the sides of a rhombus are equal. Measure $\angle AOD$ and $\angle BOC$.

What is the measures of these angles?

You will find that each of them equals 90°

Also $\angle AOB = \angle COD$ (Each pair is a vertically opposite angles)

and $\angle BOC = \angle DOA$

$\therefore$ $\angle AOB = \angle COD = \angle BOC = \angle DOA = 90°$

Thus, the diagonals of a rhombus bisect each other at right angles.

You may repeat this experiment by taking different rhombuses, you will find in each case, the diagonals of a rhombus bisect each other.

Thus, we have the following properties of a rhombus.

All sides of a rhombus are equal

Opposite angles of a rhombus are equal

The diagonals of a rhombus bisect each other at right angles.

3. Properties of a Rectangle

We know that a rectangle is a parallelogram one of whose angles is a right angle. Can you say whether a rectangle possesses all the properties of a parallelogram or not?

Yes it possesses. Let us study some more properties of a rectangle.

Draw a parallelogram ABCD in which $\angle B = 90°$.

Join AC and BD as shown in the Fig. 13.16

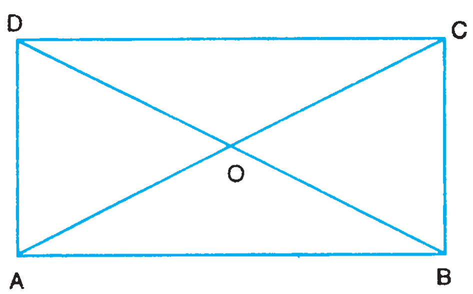

Fig. 13.16

Measure $\angle BAD$, $\angle BCD$ and $\angle ADC$, what do you find?

What are the measures of these angles?

The measure of each angle is 90°. Thus, we can conclude that

$$\angle A = \angle B = \angle C = \angle D = 90°$$

i.e., each angle of a rectangle measures 90°. Now measure the diagonals AC and BD. Do you find that AC = BD.

Now, measure AO, OC, BO and OD.

You will find that AO = OC and BO = OD.

Draw some more rectangles of different dimensions. Label them again by ABCD. Join AC and BD in each case. Let them intersect each other at O. Also measure AO, OC and BO, OD for each rectangle. In each case you will find that

The diagonals of a rectangle are equal and they bisect each other. Thus, we have the following properties of a rectangle;

The opposite sides of a rectangle are equal

Each angle of a rectangle is a right-angle.

The diagonals of a rectangle are equal.

The diagonals of a rectangle bisect each other.

4. Properties of a Square

You know that a square is a rectangle, with a pair of adjacent sides equal. Now, can you conclude from definition of a square that a square is a rectangle and possesses all the properties of a rectangle? Yes it is. Let us now study some more properties of a square.

Draw a square ABCD as shown in Fig. 13.17.

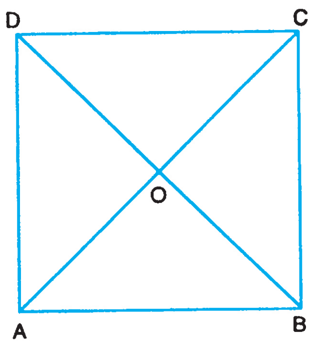

Fig 13.17

Since ABCD is a rectangle, therefore we have

(i) AB = DC, AD = BC

(ii) $\angle A = \angle B = \angle C = \angle D = 90^o$

(iii) AC = BD and AO = OC, BO = OD

But in a square we have AB = AD

∴ By property (i) we have

AB = AD = CD = BC.

Since a square is also a rhombus. Therefore, we conclude that the diagonals AC and BD of a square bisect each other at right angles.

Thus, we have the following properties of a square.

All the sides of a square are equal

Each of the angles measures 90º.

The diagonals of a square are equal.

The diagonals of a square bisect each other at right angles.

Let us study some examples to illtustrate the above properties:

Example 13.3: In Fig. 13.17, ABCD is a parallelogram. If $\angle A = 80^o$, find the measures of the remaining angles

Solution: As ABCD is a parallelogram.

$\angle A = \angle C$ and $\angle B = \angle D$

It is given that

$\angle A = 80^o$

∴ $\angle C = 80^o$

∴ AB || DC

∴ $\angle A + \angle D = 180^o$

∴ $\angle D = (180 - 80)^o = 100^o$

∴ $\angle B = \angle D = 100^o$

Hence $\angle C = 80^o$, $\angle B = 100^o$ and $\angle D$ 100^o

Fig 13.18

Example 13.4: Two adjacent angles of a rhombus are in the ratio 4 : 5. Find the measure of all its angles.

Solution: Since opposite sides of a rhombus are parallel, the sum of two adjacent angles of a rhombus is 180º.

Let the measures of two angles be $4x^o$ and $5x^o$,

Therefore, $4x + 5x = 180$

i.e. $9x = 180$

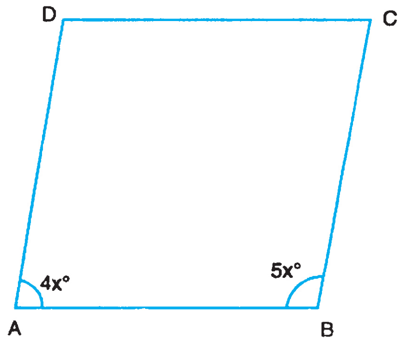

Fig 13.19

$$x = 20$$

$\therefore$ The two measures of angles are 80° and 100°.

i.e. $\angle A = 80^\circ$ and $\angle B = 100^\circ$

Since $\angle A = \angle C \Rightarrow \angle C = 100^\circ$

Also, $\angle B = \angle D \Rightarrow \angle D = 100^\circ$

Hence, the measures of angles of the rhombus are 80°, 100°, 80° and 100°.

Example 13.5: One of the diagonals of a rhombus is equal to one of its sides. Find the angles of the rhombus.

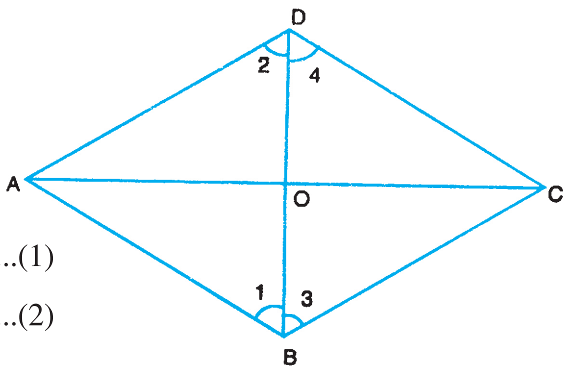

Fig 13.20

Solution: Let in rhombus, ABCD,

$$AB = AD = BD$$

$\therefore$ ΔABD is an equilateral triangle.

$\therefore$ $\angle DAB = \angle 1 = \angle 2 = 60^\circ$ ....(1)

Similarly $\angle BCD = \angle 3 = \angle 4 = 60^\circ$ ....(2)

Also from (1) and (2)

$\angle ABC = \angle B = \angle 1 + \angle 3 = 60^\circ + 60^\circ = 120^\circ$

$\angle ADC = \angle D = \angle 2 + \angle 4 = 60^\circ + 60^\circ = 120^\circ$

Hence, $\angle A = 60^\circ$, $\angle B = 120^\circ$, $\angle C = 60^\circ$ and $\angle D = 120^\circ$

Example 13.6: The diagonals of a rhombus ABCD intersect at O. If $\angle ADC = 120^\circ$ and OD = 6 cm, find

(a) $\angle OAD$

(b) side AB

(c) perimeter of the rhombus ABCD

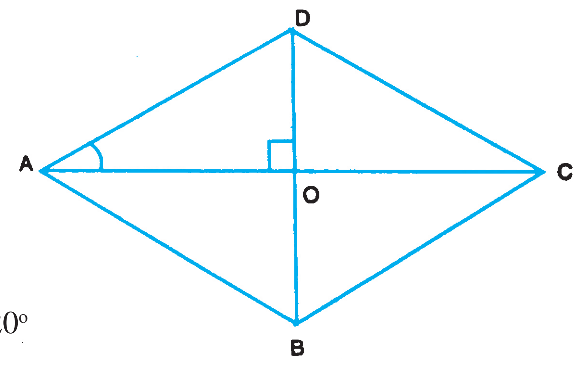

Fig 13.21

Solution: (a) Given that

$\angle ADC = 120^\circ$

i.e. $\angle ADO + \angle ODC = 120^\circ$

But $\angle ADO = \angle ODC$ $(\Delta AOD \cong \Delta COD)$

$\therefore$ $2\angle ADO = 120^\circ$

i.e. $\angle ADO = 60^\circ$ ...(i)

Also, we know that the diagonals of a rhombus bisect each that at 90°.

$\therefore \qquad \angle DOA = 90^\circ \qquad ...(ii)$

Now, in ΔDOA

$$\angle ADO + \angle DOA + \angle OAD = 180^\circ$$

From (i) and (ii), we have

$$60^\circ + 90^\circ + \angle OAD = 180^\circ$$

$$\Rightarrow \qquad \angle OAD = 30^\circ$$

(b) Now, $\angle DAB = 60^\circ$ [since $\angle OAD = 30^\circ$, similarly $\angle OAB = 30^\circ$]

$\therefore \Delta DAB$ is an equilateral triangle.

$$OD = 6 \text{ cm} \qquad \text{[given]}$$

$$\Rightarrow \qquad OD + OB = BD$$

$$6 \text{ cm} + 6 \text{ cm} = BD$$

$$\Rightarrow \qquad BD = 12 \text{ cm}$$

so, $\qquad AB = BD = AD = 12 \text{ cm}$

$$AB = 12 \text{ cm}$$

(c) Now Perimeter $= 4 \times \text{side}$

$$= (4 \times 12) \text{ cm}$$

$$= 48 \text{ cm}$$

Hence, the perimeter of the rhombus = 48 cm.

CHECK YOUR PROGRESS 13.2

1. In a parallelogram ABCD, $\angle A = 62^\circ$. Fing the measures of the other angles.
2. The sum of the two opposite angles of a parallelogram is 150°. Find all the angles of the parallelogram.
3. In a parallelogram ABCD, $\angle A = (2x + 10)^\circ$ and $\angle C = (3x - 20)^\circ$. Find the value of x.
4. ABCD is a parallelogram in which $\angle DAB = 70^\circ$ and $\angle CBD = 55^\circ$. Find $\angle CDB$ and $\angle ADB$.
5. ABCD is a rhombus in which $\angle ABC = 58^\circ$. Find the measure of $\angle ACD$.

6. In Fig. 13.22, the diagonals of a rectangle PQRS intersect each other at O. If $\angle ROQ = 40^\circ$, find the measure of $\angle OPS$.

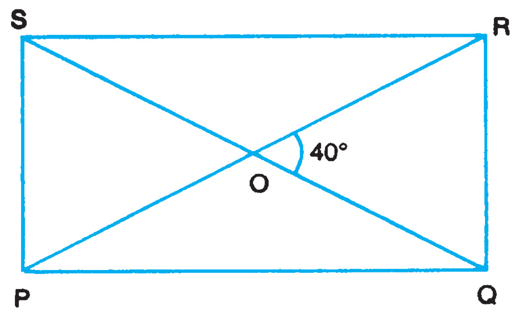

Fig 13.22

7. AC is one diagonal of a square ABCD. Find the measure of $\angle CAB$.

13.4 MID POINT THEOREM

Draw any triangle ABC. Find the mid points of side AB and AC. Mark them as D and E respectively. Join DE, as shown in Fig. 13.23.

Measure BC and DE.

What relation do you find between the length of BC and DE?

Of course, it is $DE = \frac{1}{2} BC$

Again, measure $\angle ADE$ and $\angle ABC$.

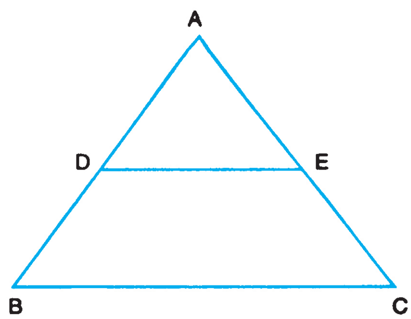

Fig 13.23

Are these angles equal?

Yes, they are equal. You know that these angles make a pair of corresponding angles. You know that when a pair of corresponding angles are equal, the lines are parallel

$\therefore \quad DE \parallel BC$

You may repeat this expreiment with another two or three triangles and naming each of them as triangle ABC and the mid point as D and E of sides AB and AC respectively.

You will always find that $DE = \frac{1}{2} BC$ and $DE \parallel BC$.

Thus, we conclude that

In a triangle the line-segment joining the mid points of any two sides is parallel to the third side and is half of it.

We can also verify the converse of the above stated result.

Draw any ΔPQR. Find the mid point of side RQ, and mark it as L. From L, draw a line LX || PQ, which intersects, PR at M.

Measure PM and MR. Are they equal? Yes, they are equal.

You may repeat with different triangles and by naming each of them as PQR and taking each time L as the mid-point of RQ and drawing a line LM || PQ, you will find in each case that RM = MP. Thus, we conclude that

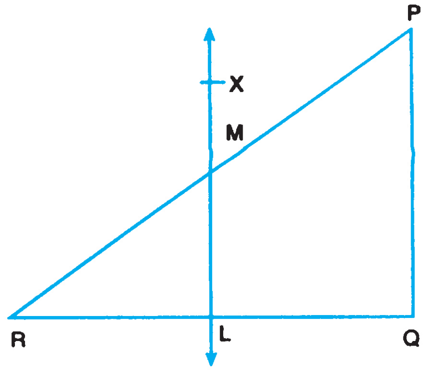

Fig 13.24

"The line drawn through the mid point of one side of a triangle parallel to the another side bisects the third side."

Example 13.7: In Fig. 13.25, D is the mid-point of the side AB of ΔABC and DE || BC. If AC = 8 cm, find AE.

Solution: In ΔABC, DE || BC and D is the mid point of AB

∴ E is also the mid point of AC

$$\text{i.e. } AE = \frac{1}{2} AC$$

$$= \left(\frac{1}{2} \times 8\right) \text{cm} \qquad [\because AC = 8 \text{ cm}]$$

$$= 4 \text{ cm}$$

Hence, $AE = 4$ cm

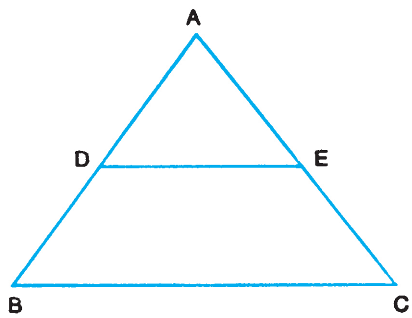

Fig 13.25

Example 13.8: In Fig. 13.26, ABCD is a trapezium in which AD and BC are its non-parallel sides and E is the mid-point of AD. EF || AB. Show that F is the mid-point of BC.

Solution: Since EG || AB and E is the mid-point of AD (considering ΔABD)

∴ G is the mid point of DB

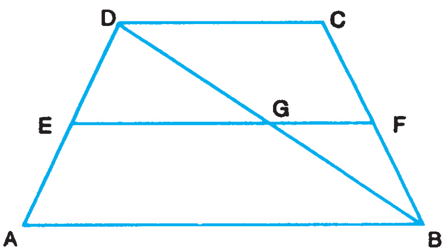

Fig 13.26

In ΔDBC, GF || DC and G is the mid-point of DB,

$\therefore$ F is the mid-point of BC.

Example 13.9: ABC is a triangle, in which P, Q and R are mid-points of the sides AB, BC and CA respectively. If AB = 8 cm, BC = 7 cm and CA = 6 cm, find the sides of the triangle PQR.

Solution: P is the mid-point of AB and R the mid-point of AC.

$\therefore$ PR || BC and PR $= \frac{1}{2}$ BC

$= \frac{1}{2} \times 7$ cm [$\because$ BC = c

$= 3.5$ cm

Similarly, PQ $= \frac{1}{2}$ AC

$= \frac{1}{2} \times 6$ cm [$\because$ AC = 6 cm]

$= 3$ cm

and QR $= \frac{1}{2}$ AB

$= \frac{1}{2} \times 8$ cm [$\because$ AB = 8 cm]

$= 4$ cm

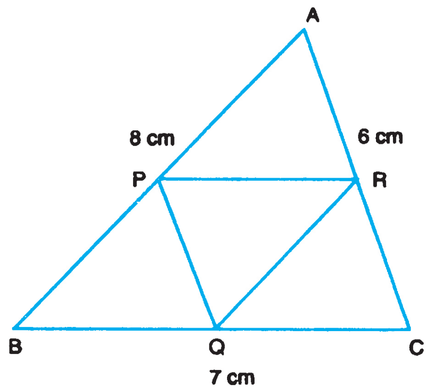

Fig 13.27

Hence, the sides of ΔPQR are PQ = 3 cm, QR = 4 cm and PR = 3.5 cm.

CHECK YOUR PROGRESS 13.3

1. In Fig. 13.28, ABC is an equilateral triangle. D, E and F are the mid-points of the sides AB, BC and CA respectively. Prove that DEF is also an equilateral triangle.

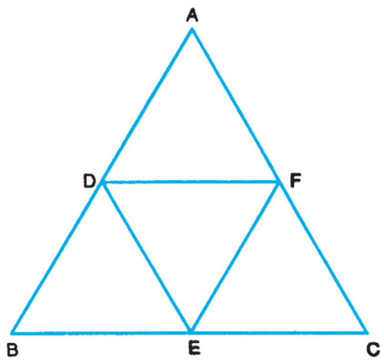

Fig. 13.28

2. In Fig. 13.29, D and E are the mid-points of the sides AB and AC respectively of a ΔABC. If BC = 10 cm; find DE.

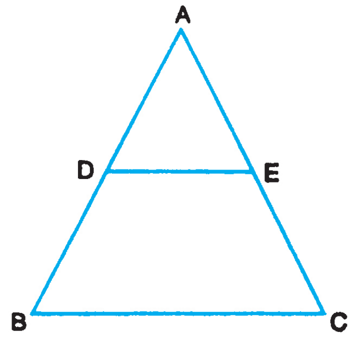

Fig. 13.29

3. In Fig. 13.30, AD is a median of the ΔABC and E is the mid-point of AD, BE is produced to meet AC at F. DG || EF, meets AC at G. If AC = 9 cm, find AF.

   [Hint: First consider ΔADG and next consider ΔCBF]

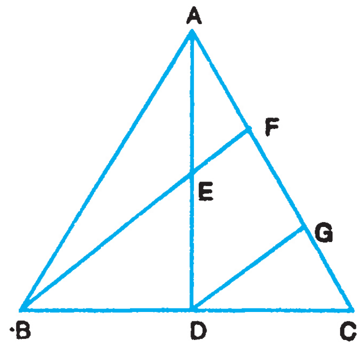

Fig. 13.30

4. In Fig. 13.31, A and C divide the side PQ of ΔPQR into three equal parts, AB||CD||QR. Prove that B and D also divide PR into three equal parts.

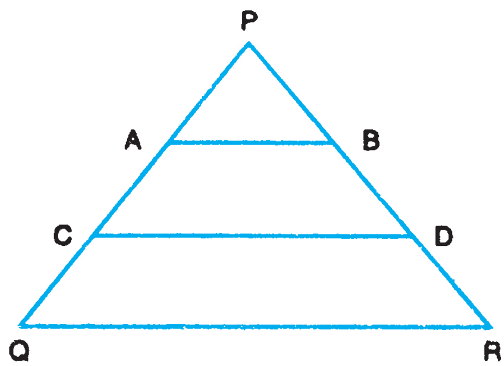

Fig. 13.31

5. In Fig. 13.32, ABC is an isosceles triangle in which AB = AC. M is the mid-point of AB and MN||BC. Show that ΔAMN is also an isosceles triangle.

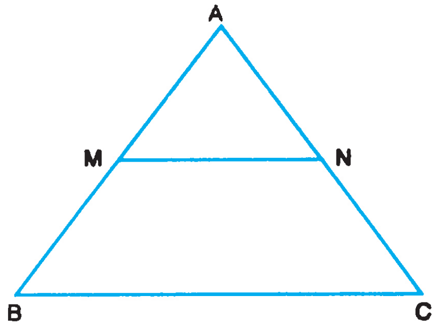

Fig. 13.32

13.5 EQUAL INTERCEPT THEORM

Recall that a line which intersects two or more lines is called a transversal. The line-segment cut off from the transversal by a pair of lines is called an intercept. Thus, in Fig. 13.33, XY is an intercept made by line *l* and m on transversal n.

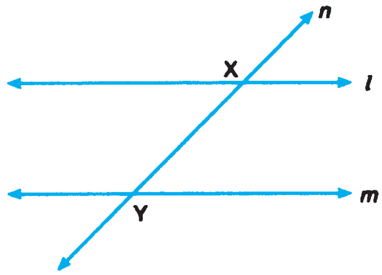

Fig. 13.33

The intercepts made by parallel lines on a transversal have some special properties which we shall study here.

Let *l* and m be two parallel lines and XY be an intercept made on the transversal "n". If there are three parallel lines and they are intersected by a transversal, there will be two intercepts AB and BC as shown in Fig. 13.34 (ii).

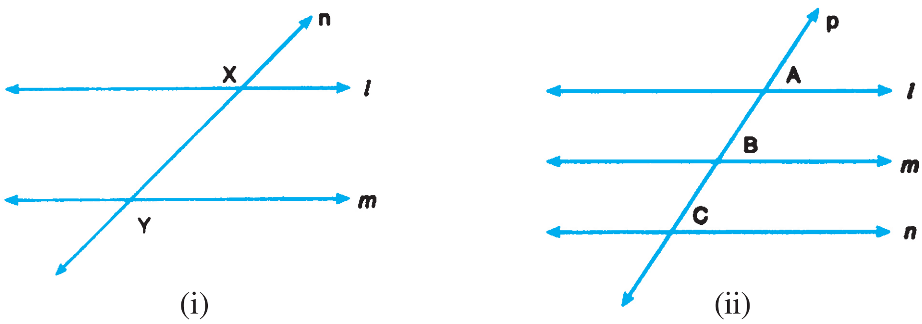

Fig. 13.34

Now let us learn an important property of intercepts made on the transversals by the parallel lines.

On a page of your note-book, draw any two transversals *l* and m intersecting the equidistant parallel lines p, q, r and s as shown in Fig. 13.35. These transversals make different intercepts. Measure the intercept AB, BC and CD. Are they equal? Yes, they are equal.

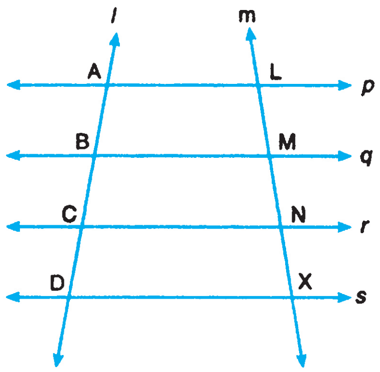

Fig. 13.35

Also, measure LM, MN and NX. Do you find that they are also equal? Yes, they are.

Repeat this experiment by taking another set of two or more equidistant parallel lines and measure their intercepts as done earlier. You will find in each case that the intercepts made are equal.

Thus, we conclude the following:

If there are three or more parallel lines and the intercepts made by them on a transversal are equal, the corresponding intercepts made on any other transversal are also equal.

Let us illustrate it by some examples: This result is known as Equal Intercept Theorm.

Example 13.10: In Fig. 13.36, p || q ||r. The transversal *l*, m and n cut them at L, M, N; A, B, C and X, Y, Z respectively such that XY = YZ. Show that AB = BC and LM = MN.

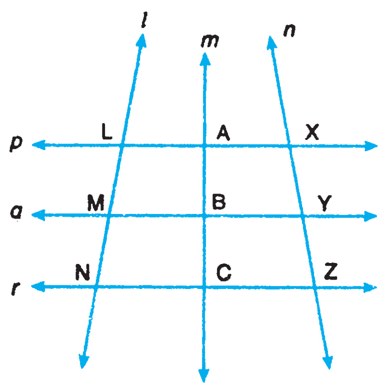

Fig. 13.36

Solution: Given that XY = YZ

∴ AB = BC (Equal Intercept theorem)

and LM = MN

Thus, the other pairs of equal intercepts are

AB = BC and LM = MN.

Example 13.11: In Fig. 13.37, *l* || m || n and PQ = QR. If XZ = 20 cm, find YZ.

Solution: We have PQ = QR

∴ By intercept theorem,

XY = YZ

Also XZ = XY + YZ

= YZ + YZ

∴ 20 = 2YZ ⇒ YZ = 10 cm

Hence, YZ = 10 cm

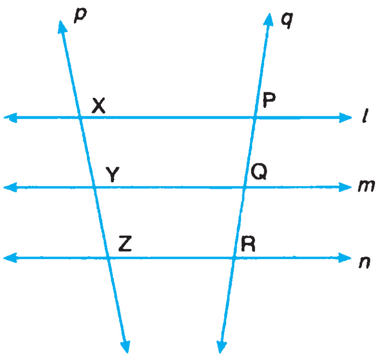

Fig. 13.37

CHECK YOUR PROGRESS 13.4

1. In Fig. 13.38, *l*, m and n are three equidistant parallel lines. AD, PQ and GH are three transversal, If BC = 2 cm and LM = 2.5 cm and AD || PQ, find MS and MN.

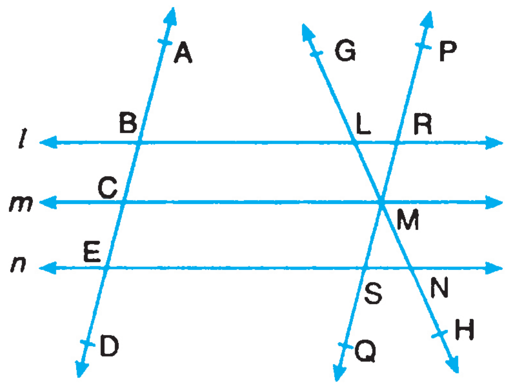

Fig. 13.38

2. From Fig. 13.39, when can you say that AB = BC and XY = YZ?

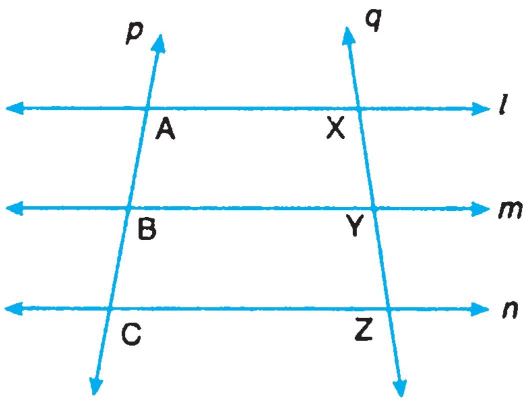

Fig. 13.39

3. In Fig. 13.40, LM = MZ = 3 cm, find XY, XP and BZ. Given that $l \parallel m \parallel n$ and PQ = 3.2 cm, AB = 3.5 cm and YZ = 3.4 cm.

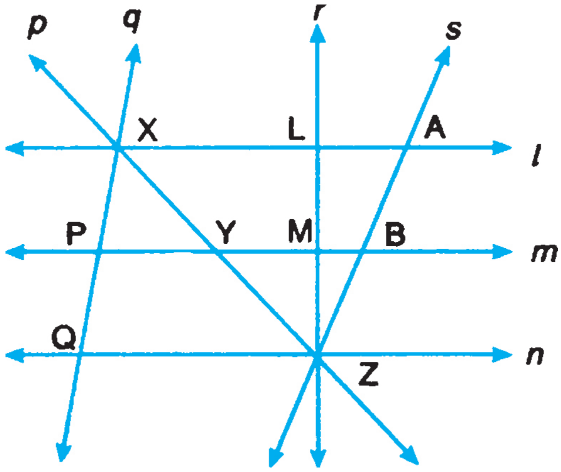

Fig. 13.40

13.6 THE DIAGONAL OF A PARALLELOGRAM AND RELATION TO THE AREA

Draw a parallelogram ABCD. Join its diagonal AC. DP ⊥ DC and QC ⊥ DC.

Consider the two triangles ADC and ACB in which the parallelogram ABCD has been divided by the diagonal AC. Because AB ∥ DC, therefore PD = QC.

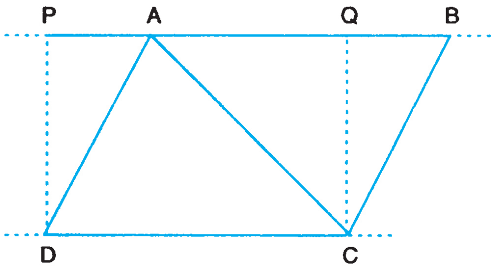

Fig. 13.41

Now, Area of $\Delta ADC = \frac{1}{2}$ DC × PD ....(i)

Area of $\Delta ACB = \frac{1}{2}$ AB × QC ....(ii)

As AB = DC and PD = QC

∴ Area (ΔADC) = Area (ΔACB)

Thus, we conclude the following:

A diagonal of a parallelogram divides it into two triangles of equal area.

13.7 PARALLELOGRAMS AND TRIANGLES BETWEEN THE SAME PARALLELS

Two parallelograms or triangles, having same or equal bases and having their other vertices on a line parallel to their bases, are said to be on the same or equal bases and between the same parallels.

We will prove an important theorem on parallelogram and their area.

Theorm: Parallelogrm on the same base (or equal bases) and between the same parallels are equal in area.

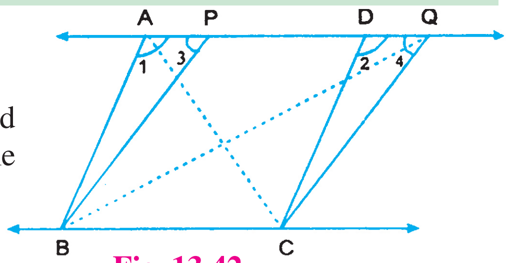

Fig. 13.42

Let us prove it logically.

Given: Parallelograms ABCD and PBCQ stand on the same base BC and between the same parallels BC and AQ.

To prove: Area (ABCD) = Area (BCQP)

we have $AB = DC$ (Opposite sides of a parallelogram)

and $BP = CQ$ (Opposite sides of a parallelogram)

$\angle 1 = \angle 2$

$\therefore \quad \Delta ABP \cong \Delta DCQ$

$\therefore$ Area (ΔABP) = Area (ΔDCQ) ...(i)

Now, Area ($||^{gm}$ ABCD) = Area (ΔABP) + Area Trapezium, BCDP) ...(ii)

Area ($||^{gm}$ BCQP) = Area (ΔDCQ) + Area Trapezium, BCDP) ...(iii)

From (i), (ii) and (iii), we get

Area ($||^{gm}$ ABCD) = Area ($||^{gm}$ BCQP)

Parallelogram on the same base (or equal bases) and between the same parallels are equal in area.

Note: $||^{gm}$ stands for parallelogram.

Result: Triangles, on the same base and between the same parallels, are equal in area.

Consider Fig. 13.42. Join the diagonals BQ and AC of the two parallelograms BCQP and ABCD respectively. We know that a diagonals of a $||^{gm}$ divides it in two triangles of equal area.

$\therefore$ Area (ΔBCQ) = Area (ΔPBQ) [Each half of $||^{gm}$ BCQP]

and Area (ΔABC) = Area (ΔCAD) [Each half of $||^{gm}$ ABCD]

$\therefore$ Area (ΔABC) = Area (ΔBCQ) [Since area of $||^{gm}$ ABCD = Area of $||^{gm}$ BCQP]

Thus we conclude the following:

Triangles on the same base (or equal bases) and between the same parallels are equal in area.

13.8 TRIANGLES ON THE SAME OR EQUAL BASES HAVING EQUAL AREAS HAVE THEIR CORRESPONDING ALTITUDES EQUAL

Recall that the area of triangle = $\frac{1}{2}$ (Base) $\times$ Altitude

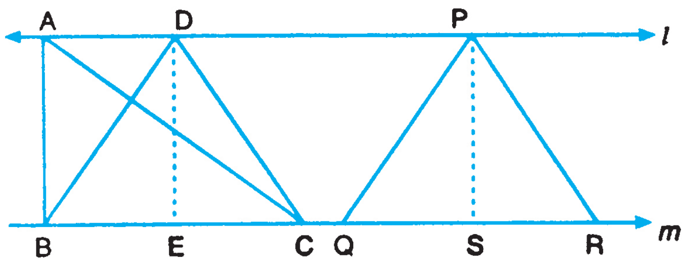

Fig. 13.43

Here BC = QR

and Area (ΔABC) = Area (ΔDBC) = Area (ΔPQR) [Given] ..(i)

Draw perpendiculars DE and PS from D and P to the line m meeting it in E and S respectively.

Now Area (ΔABC) = $\frac{1}{2}$ BC $\times$ DE

Area (ΔDBC) = $\frac{1}{2}$ BC $\times$ DE ...(ii)

and Area (ΔPQR) = $\frac{1}{2}$ QR $\times$ PS

Also, BC = QR (given) ...(iii)

From (i), (ii) and (iii), we get

$$\frac{1}{2} BC \times DE = \frac{1}{2} QR \times PS$$

or $$\frac{1}{2} BC \times DE = \frac{1}{2} BC \times PS$$

$$\therefore \quad DE = PS$$

i.e., Altitudes of ΔABC, ΔDBC and ΔPQR are equal in length.

Thus, we conclude the following:

Triangles on the same or equal bases, having equal areas have their corresponding altitudes equal.

Let us consider some examples:

Example 13.12: In Fig. 13.44, the area of parallelogram ABCD is 40 sq cm. If BC = 8 cm, find the altitude of parallelogram BCEF.

Solution: Area of $||^{gm}$ BCEF = Area of $||^{gm}$ ABCD = 40 sq cm

we know that Area ($||^{gm}$ BCEF) = EF × Altitude

or 40 = BC × Altitude of $||^{gm}$ BCEF

or 40 = BC × Altitude of $||^{gm}$ BCEF

$\therefore$ Altitude of $||^{gm}$ BCEF $= \frac{40}{8}$ cm or 5 cm.

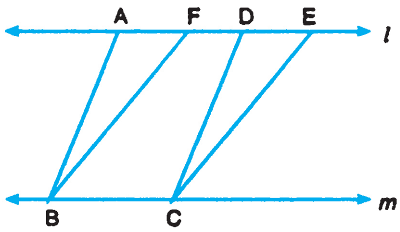

Fig. 13.44

Example 13.13: In Fig. 13.45, the area of ΔABC is given to be 18 cm^2. If the altitude DL equals 4.5 cm, find the base of the ΔBCD.

Solution: Area (ΔBCD) = Area (ΔABC) = 18 cm^2

Let the base of ΔBCD be x cm

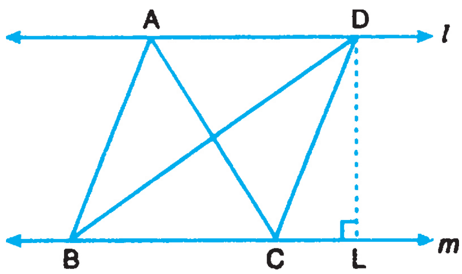

Fig. 13.45

$$\therefore \quad \text{Area of } \Delta BCD = \frac{1}{2} x \times DL$$

$$= \left(\frac{1}{2} x \times 4.5\right) cm^2$$

or $$18 = \left(\frac{9}{4} x\right)$$

$$\therefore \quad x = \left(18 \times \frac{4}{9}\right) cm = 8 \text{ cm.}$$

Example 13.14: In Fig. 13.46, ABCD and ACED are two parallelograms. If area of ΔABC equals 12 cm², and the length of CE and BC are equal, find the area of the trapezium ABED.

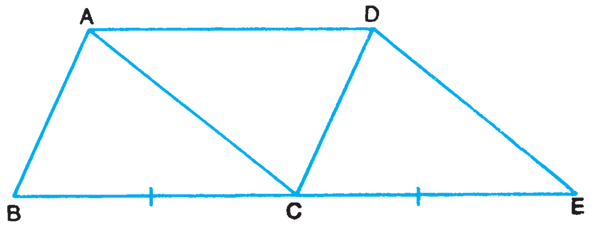

Fig. 13.46

Solution: Area (||^{gm} ABCD) = Area (||^{gm} ACED)

The diagonal AC divides the ||^{gm} ABCD into two triangles of equal area.

$\therefore$ Area (ΔBCD) = $\frac{1}{2}$ Area (||^{gm} ABCD)

$\therefore$ Area (||^{gm} ABCD) = Area (||^{gm} ACED) = 2×12 cm²

= 24 cm²

$\therefore$ Area of Trapezium ABED

= Area (ΔABC) + Area (||^{gm} ACED)

= (12 + 24) cm²

= 36 cm²

CHECK YOUR PROGRESS 13.5

1. When do two parallelograms on the same base (or equal bases) have equal areas?
2. The area of the triangle ABC formed by joining the diagonal AC of a ||^{gm} ABCD is 16 cm². Find the area of the ||^{gm} ABCD.
3. The area of ΔACD in Fig. 13.47 is 8 cm². If EF = 4 cm, find the altitude of ||^{gm} BCFE.

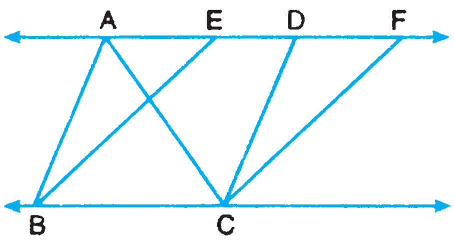

Fig. 13.47

LET US SUM UP

- A quadrilateral is a four sided closed figure, enclosing some region of the plane.
- The sum of the interior or exterior angles of a quadrilateral is equal to 360° each.
- A quadrilateral is a trapezium if its only one pair of opposite sides is parallel.
- A quadrilateral is a parallelogrm if both pairs of sides are parallel.
- In a parallelogram:
  (i) opposite sides and angles are equal.
  (ii) diagonals bisect each other.
- A parallelogram is a rhombus if its adjacent sides are equal.
- The diagonals of a rhombus bisect each other at right angle.
- A parallelogram is a rectangle if its one angle is 90°.
- The diagonals of a rectangle are equal.
- A rectangle is a square if its adjacent sides are equal.
- The diagonals of a square intersect at right angles.
- The diagonal of a parallelogram divides it into two triangles of equal area.
- Parallelogram on the same base (or equal bases) and between the same parallels are equal in area.
- The triangles on the same base (or equal bases) and between the same parallels are equal in area.
- Triangles on same base (or equal bases) having equal areas have their corrsponding altitudes equal.

TERMINAL EXERCISE

1. Which of the following are trapeziums?

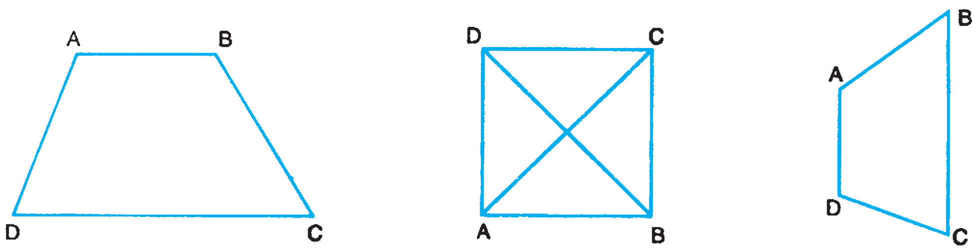

Fig. 13.48

2. In Fig. 13.49, PQ || FG || DE || BC. Name all the trapeziums in the figure.

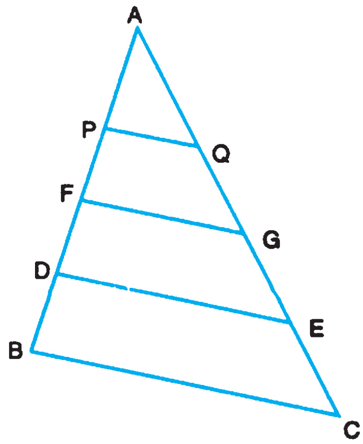

Fig. 13.49

3. In Fig. 13.50, ABCD is a parallelogram with an area of 48 cm^2. Find the area of (i) shaded region (ii) unshaded region.

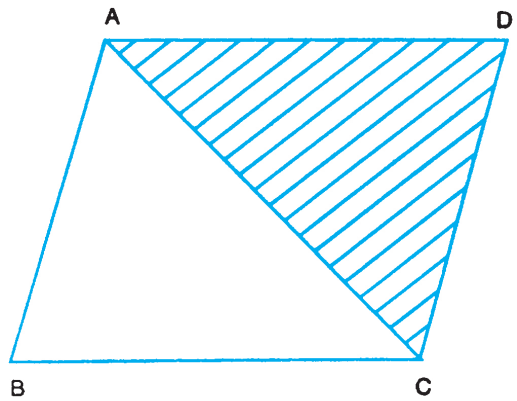

Fig. 13.49

4. Fill in the blanks in each of the following to make them true statements:

   (i) A quadrilateral is a trapezium if ....

   (ii) A quadrilateral is a parallelogram if ....

   (iii) A rectangle is a square if ...

   (iv) the diagonals of a quadrilateral bisect each other at right angle. If none of the angles of the quadrilateral is a right angle, it is a ...

   (v) The sum of the exterior angles of a quadrilateral is ...

5. If the angles of a quadrilateral are $(x - 20)^o$, $(x + 20)^o$, $(x - 15)^o$ and $(x + 15)^o$, find x and the angles of the quadrilateral.

6. The sum of the opposite angles of a parallelograms is 180^o. What type of a parallelogram is it?

7. The area of a ΔABD in Fig. 13.51 is 24 cm^2. If DE = 6 cm, and AB || CD, BD || CE, AE || BC, find

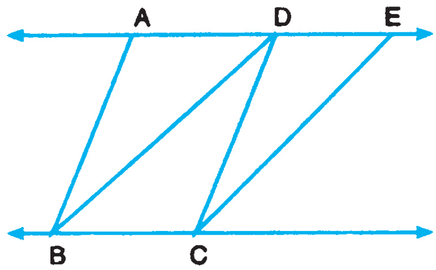

Fig. 13.51

(i) Altitude of the parallelogram BCED.

(ii) Area of the parallelogram BCED.

8. In Fig. 13.52, the area of parallelogram ABCD is 40 cm^2. If EF = 8 cm, find the altitude of ΔDCE.

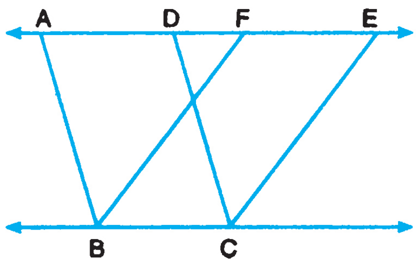

Fig. 13.52

ANSWERS TO CHECK YOUR PROGRESS

13.1

1. (i) Rectangle (ii) Trapezium (iii) Rectangle (iv) Parallelogram
   (v) Rhombus (vi) Square
2. (i) True (ii) False (iii) True (iv) True
   (v) True (vi) True (vii) False (viii) False
   (ix) False (x) False
3. 90°
4. 60°, 84°, 84° and 132°
5. Other pair of opposite angles will also be supplementary.

13.2

1. $\angle B = 118^\circ$, $\angle C = 62^\circ$ and $\angle D = 118^\circ$
2. $\angle A = 105^\circ$, $\angle B = 75^\circ$, $\angle C = 105^\circ$ and $\angle D = 75^\circ$

3. 30
4. $\angle CDB = 55^\circ$ and $\angle ADB = 55^\circ$
5. $\angle ACD = 61^\circ$
6. $\angle OPS = 70^\circ$ 7. $\angle CAB = 45^\circ$

13.3

2. 5 cm
3. 3 cm

13.4

1. MS = 2 cm and MN = 2.5 cm
2. 1, m and n are three equidistant parallel lines
3. XY = 3.4 cm, XP = 3.2 cm and BZ = 3.5 cm

13.5

1. When they are lying between the same parallel lines
2. 32 cm^2
3. 4 cm

ANSWERS TO TERMINAL EXERCISE

1. (i) and (iii)
2. PFGQ, FDEG, DBCE, PDEQ, FBCG and PBCQ
3. (i) 24 cm^2 (ii) 24 cm^2
4. (i) any one pair of opposite sides are parallel.
   (ii) both pairs of opposite sids are parallel
   (iii) pair of adjacent sides are equal
   (iv) rhombus
   (v) 360°
5. $x = 90^\circ$, angles are 70°, 110°, 75° and 105° respectively.
6. Rectangle.
7. (i) 8 cm (ii) 48 cm^2
8. 5 cm