211en11

11

CONGRUENCE OF TRIANGLES

You might have observed that leaves of different trees have different shapes, but leaves of the same tree have almost the same shape. Although they may differ in size. The geometrical figures which have same shape and same size are called congruent figures and the property is called congruency.

In this lesson you will study congruence of two triangles, some relations between their sides and angles in details.

OBJECTIVES

After studying this lesson, you will be able to

- *verify and explain whether two given figures are congruent or not.*
- *state the criteria for congruency of two triangles and apply them in solving problems.*
- *prove that angles opposite to equal sides of a triangle are equal.*
- *prove that sides opposite to equal angles of a triangle are equal.*
- *prove that if two sides of triangle are unequal, then the longer side has the greater angle opposite to it.*
- *state and verify inequalities in a triangle involving sides and angles.*
- *solve problems based on the above results.*

EXPECTED BACKGROUND KNOWLEDGE

- Recognition of plane geometrical figures
- Equality of lines and angles
- Types of angles
- Angle sum property of a triangle
- Paper cutting and folding.

11.1 CONCEPT OF CONGRUENCE

In our daily life you observe various figures and objects. These figures or objects can be categorised in terms of their shapes and sizes in the following manner.

(i) Figures, which have different shapes and sizes as shown in Fig. 11.1

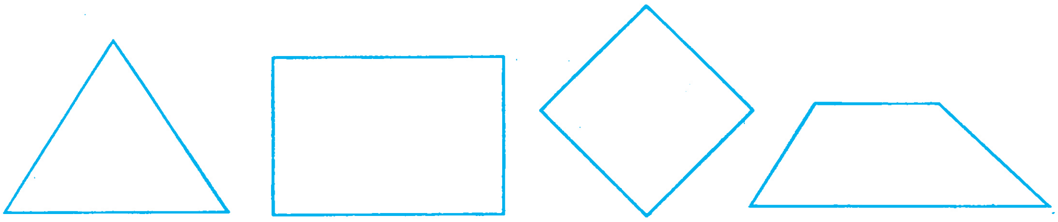

Fig. 11.1

(ii) Objcts, which have same shpaes but different sizes as shown in Fig. 11.2

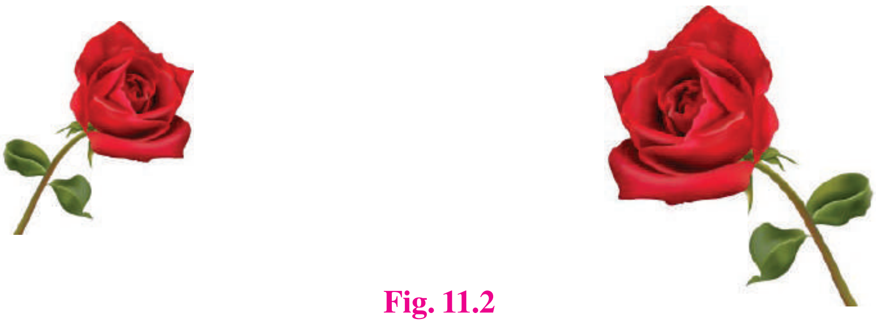

Fig. 11.2

(iii) Two one-rupee coins.

Fig. 11.3

(iv) Two postage stamps on post cards

Fig. 11.4

(v) Two photo prints of same size from the same negative.

Fig. 11.5

We will deal with the figures which have same shapes and same sizes.

Two figures, which have the same shape and same size are called congruent figures and this property is called congruence.

11.1.1. Activity

Take a sheet of paper, fold it in the middle and keep a carbon (paper) between the two folds. Now draw a figure of a leaf or a flower or any object which you like, on the upper part of the sheet. You will get a carbon copy of it on the sheet below.

The figure you drew and its carbon copy are of the same shape and same size. Thus, these are congruent figures. Observe a butterfly folding its two wings. These appear to be one.

11.1.2 Criteria for Congruence of Some Figures

Congruent figures, when palced one over another, exactly coincide with one another or cover each other. In other words, two figures will be congruent, if parts of one figure are equal to the corresponding parts of the other. For example :

(1) Two line - segments are congruent, when they are of equal length.

Fig. 11.6

(2) Two squares are congruent if their sides are equal.

Fig. 11.7

(3) Two circles are congruent, if their radii are equal, implying their circumferences are also equal.

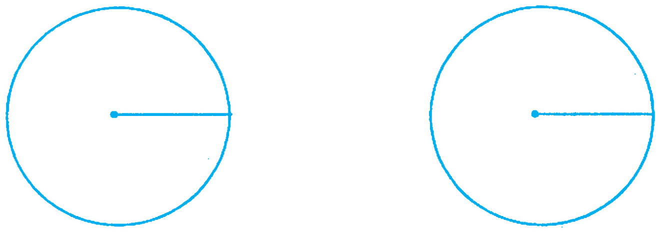

Fig. 11.8

11.2 CONGRUENCE OF TRIANGLES

Triangle is a basic rectilinear figure in geometry, having minimum number of sides. As such congruence of triangles plays a very important role in proving many useful results. Hence this needs a detailed study.

Two triangles are congruent, if all the sides and all the angles of one are equal to the corresponding sides and angles of other.

For example, in triangles PQR and XYZ in Fig. 11.9

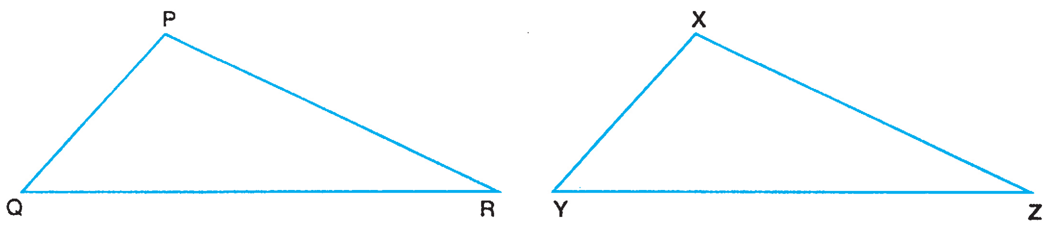

Fig. 11.9

$$PQ = XY, PR = XZ, QR = YZ$$

$$\angle P = \angle X, \angle Q = \angle Y \text{ and } \angle R = \angle Z$$

Thus we can say

Δ PQR is congruent to Δ XYZ and we write

$$\Delta PQR \cong \Delta XYZ$$

Relation of congruence between two triangles is always written with corresponding or matching parts in proper order.

Here $\Delta PQR \cong \Delta XYZ$

also means P corresponds to X, Q corresponds to Y and R corresponds to Z.

This congruence may also be written as Δ QRP ≅Δ YZX whichmeans, Q corresponds to Y, R corresponds to Z and P corresponds to X. It also means corresponding parts, (elements) are equal, namely

$$QR = YZ, RP = ZX, QP = YX, \angle Q = \angle Y, \angle R = \angle Z$$

and $\angle P = \angle X$

This congruence may also be written as

$$\Delta RPQ \cong \Delta ZXY$$

but NOT as $\Delta PQR \cong \Delta YZX$.

Or NOT as $\Delta PQR \cong \Delta ZXY$.

11. 3 CRITERIA FOR CONGRUENCE OF TRIANGLES

In order to prove, whether two triangles are congruent or not, we need to know that all the six parts of one triangle are equal to the corresponding parts of the other triangle. We shall now learn that it is possible to prove the congruence of two triangles, even if we are able to know the equality of three of their corresponding parts.

Consider the triangle ABC in Fig. 11.10

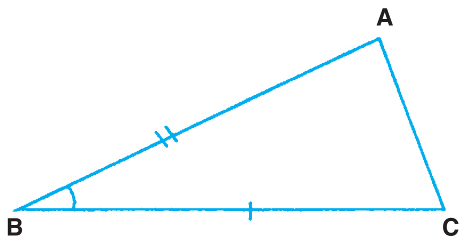

Fig. 11.10

Construct another triangle PQR such that QR = BC, ∠Q = ∠B and PQ = AB. (See Fig. 11.11)

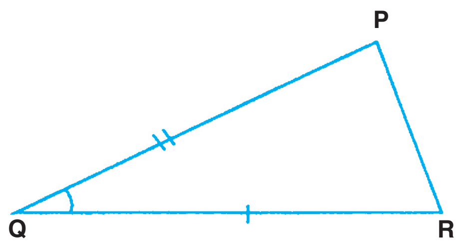

Fig. 11.11

If we trace or cut out triangle ABC and place it over triangle PQR. we will observe that one covers the other exactly. Thus, we may say that they are congruent.

Alternatively we can also measure the remaining parts, and observe that

$AC = PR, \angle A = \angle P$ and $\angle C = \angle R$

showing that $\Delta PQR \cong \Delta ABC$.

It should be noted here that in constructing ΔPQR congruent to ΔABC we used only two parts of sides $PQ = AB$, $QR = BC$ and the included angle between them $\angle Q = \angle B$.

This means that equality of these three corresponding parts results in congruent triangles. Thus we have

Criterion 1 : If any two sides and the included angle of one triangle are equal to the corresponding sides and the included angle of the other triangle, the two triangles are congruent.

This criterion is referred to as SAS (Side Angle Side).

Again, consider ΔABC in Fig. 11.12

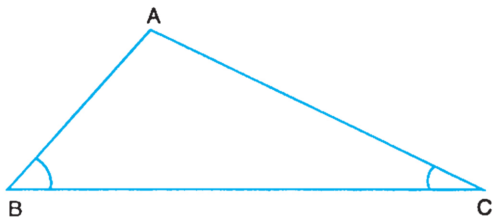

Fig. 11.12

Construct another ΔPQR such that, $QR = BC$, $\angle Q = \angle B$ and $\angle R = \angle C$. (See Fig. 11.13)

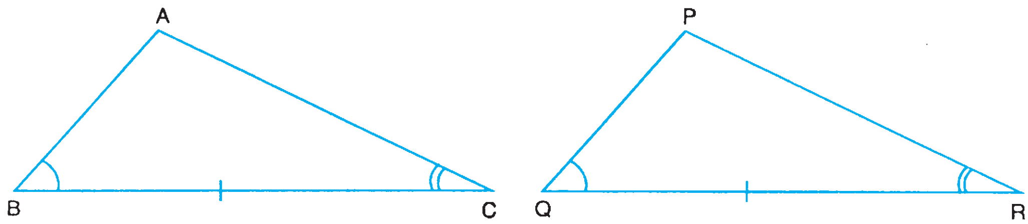

Fig. 11.13

By superimposition or by measuring the remaining corresponding parts, we observe that $\angle P = \angle A$, $PQ = AB$ and $PR = AC$ establishing that $\Delta PQR \cong \Delta ABC$, which again means that equality of the three corresponding parts (two angles and the inluded side) of two triangles results in congruent triangles.

We also know that the sum of the three angles of a triangle is 180^o, as such if two angles of one triangle are equal to the corresponding angles of another triangle, then the third angles will also be equal. Thus instead of included side we may have any pair of corresponding sides equal. Thus we have

Criterion 2 : If any two angles and one side of a triangle are equal to corresponding angles and the side of another triangle, then the two triangles are congruent.

This criterion is referred to as ASA or AAS (Angle Side Angle or Angle Angle Side)

11.3.1 Activity

In order to explore another criterion we again take a triangle ABC (See Fig. 11.14)

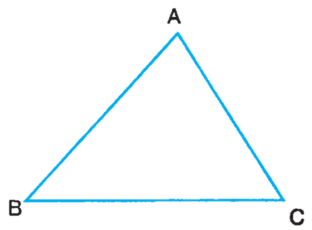

Fig. 11.14

Now take three thin sticks equal in lengths to sides AB, BC and CA of Δ ABC. Place them in any order to form Δ PQR or Δ P′Q′R′ near the Δ ABC (Fig. 11.15)

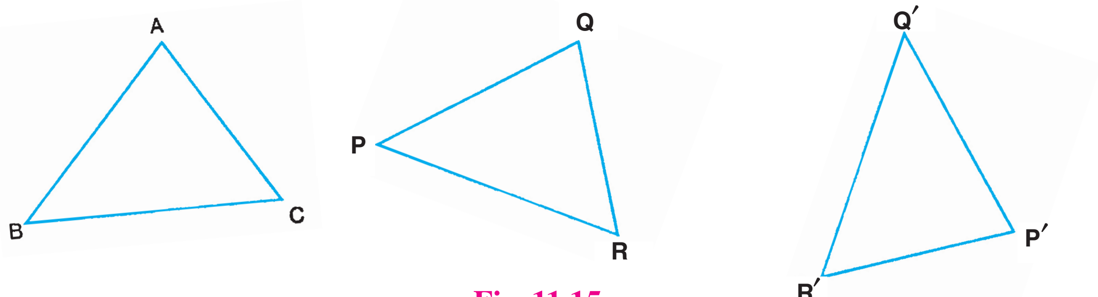

Fig. 11.15

By measuring the corresponding angles. we find that, $\angle P = \angle P' = \angle A$, $\angle Q = \angle Q' = \angle B$ and $\angle R = \angle R' = \angle C$, establishing that

$$\Delta\ PQR \cong \Delta\ P'Q'R' \cong \Delta\ ABC$$

which means that equality of the three corresponding sides of two triangles results in congruent triangles. Thus we have

Criterion 3 : If the three sides of one trianle are equal to the corresponding sides of another triangle, then the two triangles are congruent.

This is referred to as SSS (Side, Side, Side), criterion.

Similarly, we can establish one more criterion which will be applicable for two right trangles only.

Criterion 4 : If the hypotenuse and a side of one right triangle are respectively equal to the hypotenuse and a side of another right triangle, then the two triangles are congruent.

This criterion is referred to as RHS (Right Angle Hypotenuse Side).

Using these criteria we can easily prove, knowing three corresponding parts only, whether two triangles are congruent and establish the equality of remaining corresponding parts.

Example 11.1 : In which of the following criteria, two given triangles are **NOT** congruent.

(a) All corresponding sides are equal

(b) All corresponding angles are equal

(c) All corresponding sides and their included angles are equal

(d) All corresponding angles and any pair of corresponding sides are equal.

Ans. (b)

Example 11.2 : Two rectilinear figures are congruent if they have

(a) All corresponding sides equal

(b) All corresponding angles equal

(c) The same area

(d) All corresponding angles and all corresponding sides equal.

Ans. (d)

Example 11.3 : In Fig. 11.16, PX and QY are perpendicular to PQ and PX = QY. Show that AX = AY.

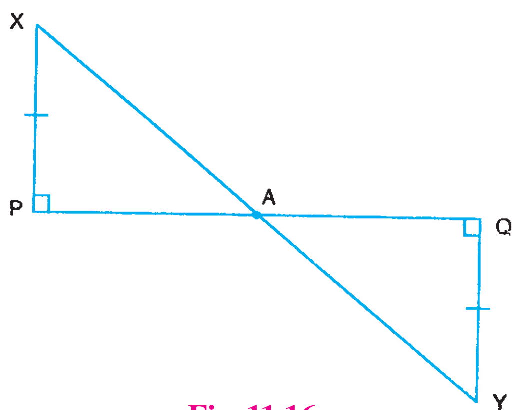

Fig. 11.16

Solution :

In Δ PAX and Δ QAY,

$\angle XPA = \angle YQA$ (Each is 90^o)

$\angle PAX = \angle QAY$ (Vertically opposite angles) and $\quad PX = QY$

$\therefore\ \Delta PAX \cong \Delta QAY \qquad$ (AAS)

$\therefore\ AX = AY.$

Example 11.4 : In Fig. 11.17, ΔABC is right triangle in which $\angle B = 90^0$ and D is the mid point of AC.

Prove that $BD = \frac{1}{2} AC.$

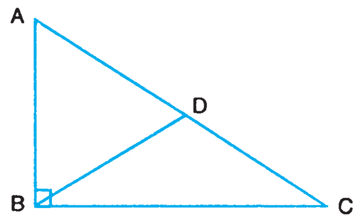

Fig. 11.17

Solution : Produce BD to E such that BD = DE. Join CE

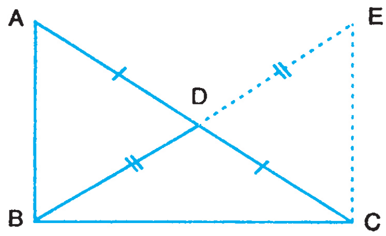

Fig.. 11.18

In ΔADB and ΔCDE,

$AD = CD \qquad$ (D being and point of AC)

$DB = DE \qquad$ (By construction)

and $\quad \angle ADB = \angle CDE \qquad$ (Vertically opposite angles)

$\therefore \quad \Delta ADB \cong \Delta CDE \qquad$ (i)

$\therefore \quad AB = EC$

Also $\quad \angle DAB = \angle DCE$

But they make a pair of alternate angles

$\therefore\ $ AB is parallel to EC

$\therefore \quad \angle ABC + \angle ECB = 180^0 \qquad$ (Pair of interior angles)

$\therefore \quad \angle 90^0 + \angle ECB = 180^0$

$\therefore \quad \angle ECB = 180^0 - 90^0 = 90^0$

Now in Δ ABC and Δ ECB,

	AB = EC	(From (i) above)
	BC = BC	(Common)
and	$\angle ABC = \angle ECB$	(Each 90^0)
$\therefore$	$\Delta ABC \cong \Delta ECB$	
$\therefore$	AC = EB	
But	$BD = \frac{1}{2} EB$	
$\therefore$	$BD = \frac{1}{2} AC$	

CHECK YOUR PROGRESS 11.1

1. In Δ ABC (Fig. 11.19) if $\angle B = \angle C$ and AD $\perp$ BC, then $\Delta ABD \cong \Delta ACD$ by the criterion.

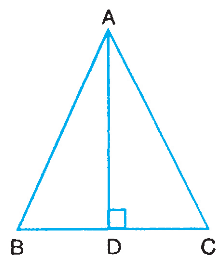

Fig. 11.19

(a) RHS (b) ASA

(c) SAS (d) SSS

2. In Fig. 11.20, $\Delta ABC \cong \Delta PQR$. This congruence may also be written as

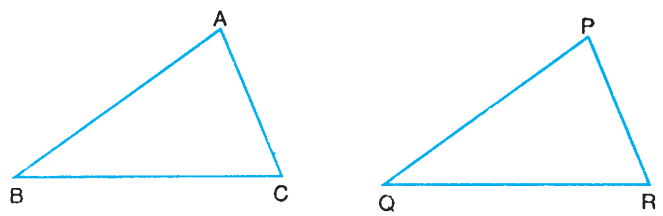

Fig. 11.20

(a) Δ BAC $\cong \Delta$ RPQ (b) Δ BAC $\cong \Delta$ QPR

(c) Δ BAC $\cong \Delta$ RQP (d) Δ BAC $\cong \Delta$ PRQ.

3. In order that two given triangles are congruent, along with equality of two corresponding angles we must know the equality of :

(a) No corresponding side

(b) Minimum one corresponding side

(c) Minimum two corresponding sides

(d) All the three corresponding sides

4. Two triangles are congruent if ....

(a) All three corresponding angles are equal

(b) Two angles and a side of one are equal to two angles and a side of the other.

(c) Two angles and a side of one are equal to two angles and the corresponding side of the other.

(d) One angle and two sides of one are equal to one angle and two sides of the other.

5. In Fig. 11.21, $\angle B = \angle C$ and AB = AC. Prove that Δ ABE $\cong \Delta$ ACD. Hence show that CD = BE.

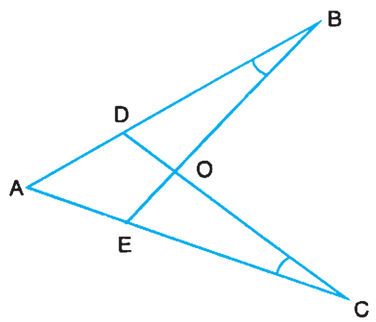

Fig. 11.21

6. In Fig. 11.22, AB is parallel to CD. If O is the mid-point of BC, show that it is also the mid-point of AD.

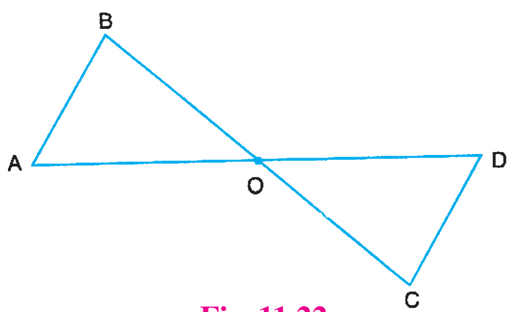

Fig. 11.22

7. In Δ ABC (Fig. 11.23), AD is ⊥ BC, BE is ⊥ AC and AD = BE. Prove that AE = BD.

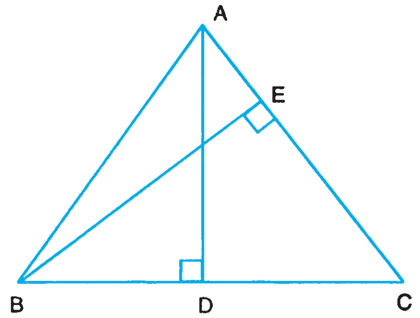

Fig. 11.23

8. From Fig. 11.24, show that the triangles are congruent and make pairs of equal angles.

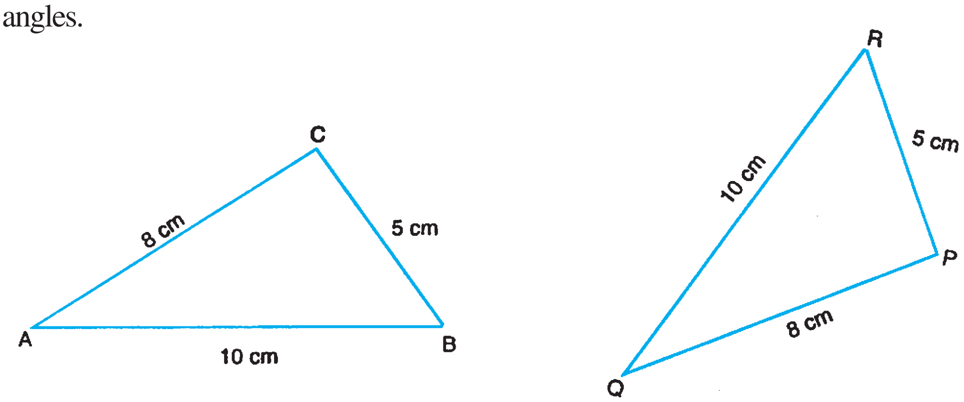

Fig. 11.24

11.4 ANGLES OPPOSITE TO EQUAL SIDES OF A TRIANGLE AND VICE VERSA

Using the criteria for congruence of triangles, we shall now prove some important theorems.

Theorem : The angles opposite to equal sides of a triangle are equal.

Given : A triangle ABC in which AB = AC.

To prove : ∠B = ∠C.

Construction : Draw bisector of ∠BAC meeting BC at D.

Proof : In Δ ABD and Δ ACD,

AB = AC (Given)

∠BAD = ∠CAD (By construction)

and AD = AD (Common)

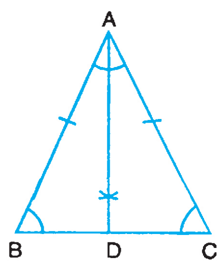

Fig. 11.25

$\Delta ABD \cong \Delta ACD$ (SAS)

Hence $\angle B = \angle C$ (Corresponding parts of congruent triangles)

The converse of the above theorem is also true. We prove it as a theorem.

11.4.1 The sides opposite to equal angles of a triangle are equal

Given : A triangle ABC in which $\angle B = \angle C$

To prove : AB = AC

Construction : Draw bisector of $\angle BAC$ meeting BC at D.

Proof : In Δ ABD and Δ ACD,

$\angle B = \angle C$ (Given)

$\angle BAD = \angle CAD$ (By construction)

and AD = AD (Common)

$\Delta ABD \cong \Delta ACD$ (SAS)

Hence AB = AC (c.p.c.t)

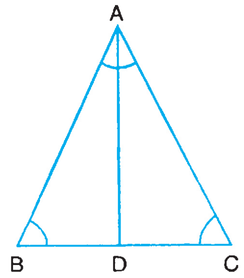

Fig. 11.26

Hence the theorem.

Example 11.5 : Prove that the three angles of an equilateral triangle are equal.

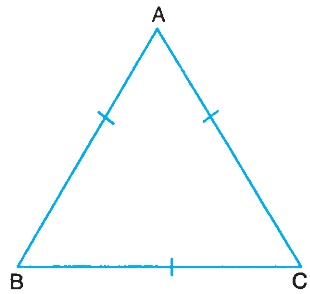

Fig. 11.27

Solution :

Given : An equilateral Δ ABC

To prove : $\angle A = \angle B = \angle C$

Proof : AB = AC (Given)

$\therefore$ $\angle C = \angle B$ (Angles opposite equal sides) ...(i)

Also AC = BC (Given)

$\therefore$ $\angle B = \angle A$ ...(ii)

From (i) and (ii),

$\angle A = \angle B = \angle C$

Hence the result.

Example 11.6 : ABC is an isosceles triangle in which AB = AC

(Fig. 11.28), If BD $\perp$ AC and CE $\perp$ AB, proe that BD = CE.

Notes

Solution : In ΔBDC and ΔCEB

∠BDC = ∠CEB (Measure of each is 90°)

∠DCB = ∠EBC (Angles opposite equal sides of a triangle)

and BC = CB (Common)

∴ Δ BDC ≅ Δ CEB (AAS)

Hence BD = CE (c.p.c.t.)

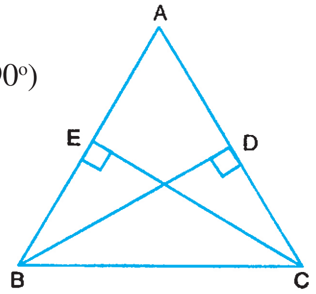

Fig. 11.28

This result can be stated in the following manner:

Perpendiculars (altitudes) drawn to equal sides, from opposite vertices of an isosceles triangle are equal.

The result can be extended to an equilateral triangle after which we can say that all the three altitudes of an equilateral triangle are equal.

Example : 11.7 : In Δ ABC (Fig. 11.29), D and E are mid-points of AC and AB respectively.

If AB = AC, then prove that BD = CE.

Solution : $BE = \frac{1}{2} AB$

and $CD = \frac{1}{2} AC$

∴ BE = CD ...(i)

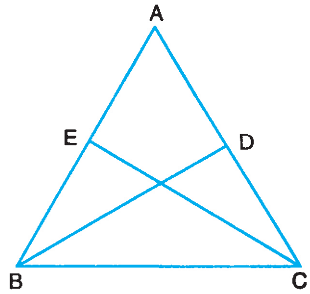

Fig. 11.29

In Δ BEC and Δ CDB,

BE = CD [By (i)]

BC = CB (Common)

and ∠EBC = ∠DCB (∵ AB = AC)

∴ Δ BEC ≅ Δ CDB (SAS)

Hence, CE = BD (c.p.c.t)

Example 11.8 : In Δ ABC (Fig. 11.30) AB = AC and ∠DAC = 124°; find the angles of the triangle.

Solution ∠BAC = 180° – 124° = 56°

∠B = ∠C

(Angles opposite to equal sides of a triangle)

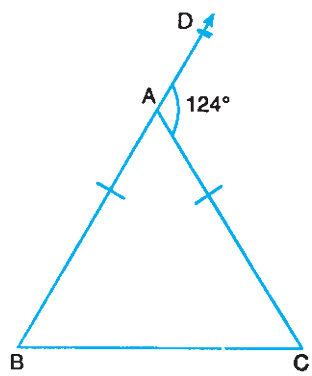

Fig. 11.30

Also $\angle B + \angle C = 124^{\circ}$

$$\angle B = \angle C = \frac{124^0}{2} = 62^0$$

Hence $\angle A = 56^{\circ}, \angle B = 62^{\circ}$, and $\angle C = 62^{\circ}$

CHECK YOUR PROGRESS 11.2

1. In Fig. 11.31, PQ = PR and SQ = SR. Prove that $\angle PQS = \angle PRS$.

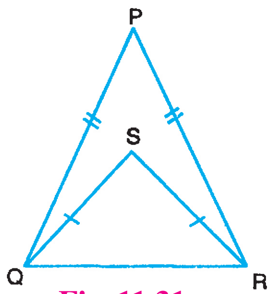

Fig. 11.31

2. Prove that ΔABC is an isosceles triangle, if the altitude AD bisects the base BC (Fig. 11.32).

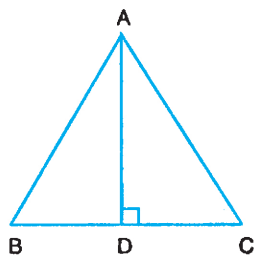

Fig. 11.32

3. If the line l in Fig. 11.33 is parallel to the base BC of the isosceles ΔABC, find the angles of the triangle.

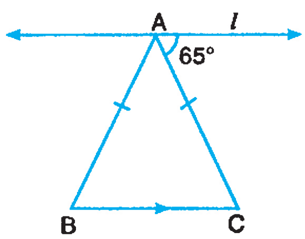

Fig. 11.33

4. ΔABC is an isosceles triangle such that AB = AC. Side BA is produced to a point D such that AB = AD. Prove that $\angle BCD$ is a right angle.

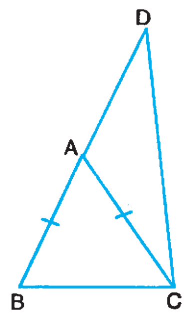

Fig. 11.34

5. In Fig. 11.35. D is the mid point of BC and perpendiculars DF and DE to sides AB and AC respectively are equal in length. Prove that ΔABC is an isosceles triangle.

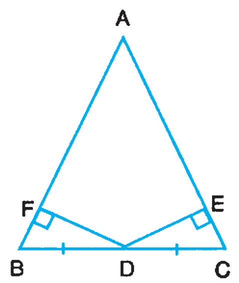

Fig. 11.35

6. In Fig. 11.36, PQ = PR, QS and RT are the angle bisectors of ∠Q and ∠R respectively. Prove that QS = RT.

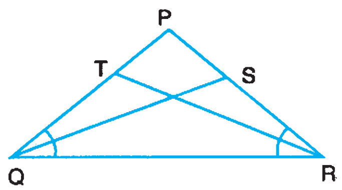

Fig. 11.36

7. ΔPQR and ΔSQR are isosceles triangles on the same base QR (Fig. 11.37). Prove that ∠PQS = ∠PRS.

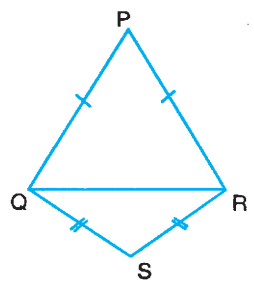

Fig. 11.37

8. In ΔABC, AB = AC (Fig. 11.38). P is a point in the interior of the triangle such that ∠ABP = ∠ACP . Prove that AP bisects ∠BAC.

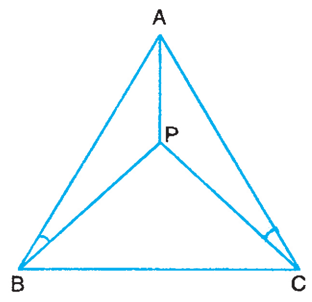

Fig. 11.38

11.5 INEQUALITIES IN A TRIANGLE

We have learnt the relationship between sides and angles of a triangle when they are equal. We shall now study some relations among sides and angles of a triangle, when they are unequal.

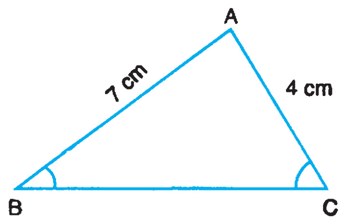

Fig. 11.39

In Fig. 11.39, triangle ABC has side AB longer than the side AC. Measure ∠B and ∠C. You will find that these angles are not equal and ∠C is greater than ∠B. If you repeat this experiment, you will always find that this observation is true. This can be proved easily, as follows.

11.5.1 Theorem

If two sides of a triangle are unequal, then the longer side has the greater angle opposite to it.

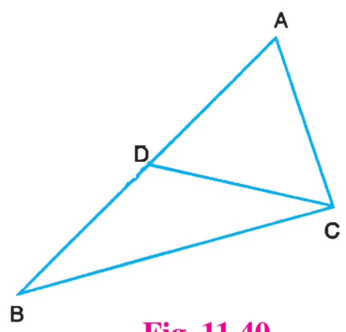

Fig. 11.40

Given. A triangle ABC in which AB > AC.

To prove. ∠ACB > ∠ABC

Construction. Make a point D on the side AB such that AD = AC and join DC.

Proof: In ΔACD,

AD = AC

∴ ∠ACD = ∠ADC (Angles opposite equal sides)

But $\angle ADC > \angle ABC$

(Exterior angle of a triangle is greater than opposite interior angle)

Again $\angle ACB > \angle ACD$ (Point D lies in the interior of the $\angle ACB$).

$\therefore$ $\angle ACB > \angle ABC$

What can we say about the converse of this theorem. Let us examine.

In ΔABC, (Fig. 11.41) compare $\angle C$ and $\angle B$. It is clear that $\angle C$ is greater than $\angle B$. Now compare sides AB and AC opposite to these angles by measuring them. We observe that AB is longer than AC.

Again compare $\angle C$ and $\angle A$ and measure sides AB and BC opposite to these angles. We observe that $\angle C > \angle A$ and AB > BC; i.e. side opposite to greater angle is longer.

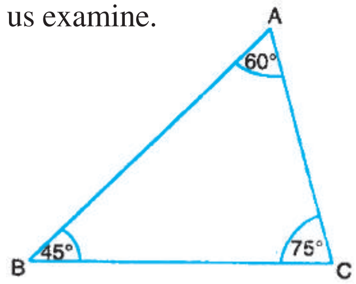

Fig. 11.41

Comparing $\angle A$ and $\angle B$, we observe a similar result. $\angle A > \angle B$ and BC > AC; i.e. side opposite to greater angle is longer.

You can also verify this property by drawing any type of triangle, a right triangle or an obtuse triangle.

Measure any pair of angles in a triangle. Compare them and then compare the sides opposite to them by measurement. You will find the above result always true, which we state as a property.

In a triangle, the greater angle has longer side opposite to it.

Observe that in a triangle if one angle is right or an obtuse then the side opposite to that angle is the longest.

You have already learnt the relationship among the three angles of a triangle i.e., the sum of the three angles of a triangle is 180°. We shall now study whether the three sides of a triangle are related in some way.

Draw a triangle ABC.

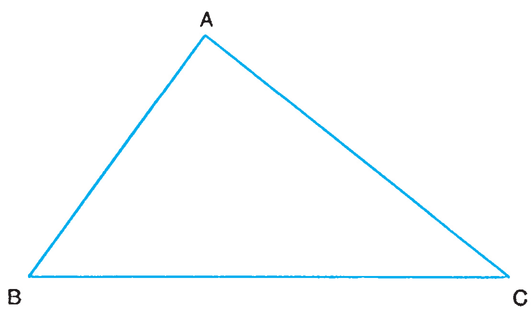

Fig. 11.42

Measure its three sides AB, BC and CA.

Now find the sum of different pairs AB+BC, BC+CA, and CA+AB separately and compare each sum of a pair with the third side, we observe that

(i) AB + BC>CA

(ii) BC + CA > AB and

(iii) CA + AB > BC

Thus we conclude that

Sum of any two sides of a triangle is greater than the third side.

ACTIVITY

Fix three nails P, Q and R on a wooden board or any surface.

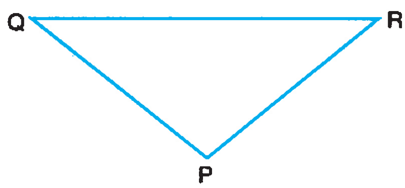

Fig. 11.43

Take a piece of thread equal in length to QR and another piece of thread equal in length (QP + PR). Compare the two lengths, you will find that the length corresponding to (QP + PR) > the length corresponding to QR confirming the above property.

Example 11.9 : In which of the following four cases, is construction of a triangle possible from the given measurements

(a) 5 cm, 8 cm and 3 cm

(b) 14 cm, 6 cm and 7 cm

(c) 3.5 cm, 2.5 cm and 5.2 cm

(d) 20 cm, 25 cm and 48 cm.

Solution. In (a) $5 + 3 \ngtr 8$, in (b) $6 + 7 \ngtr 14$

in (c) $3.5 + 2.5 > 5.2$, $3.5 + 5.2 > 2.5$ and $2.5 + 5.2 > 3.5$ and

in (d) $20 + 25 \ngtr 48$.

Ans. (c)

Example 11.10 : In Fig. 11.44, AD is a median of ΔABC. Prove that $AB + AC > 2AD$.

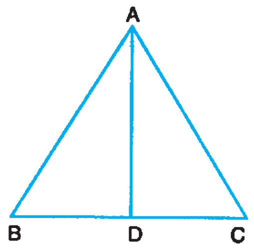

Fig. 11.44

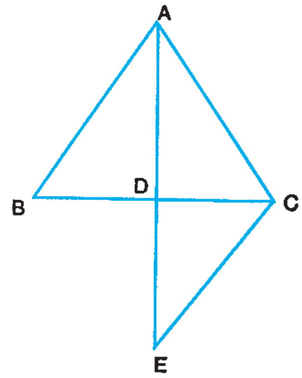

Fig. 11.45

Solution: Produce AD to E such that AD = DE and join C to E.

Consider ΔABD and ΔECD

Here, $BD = CD$

$\angle ADB = \angle EDC$

and $AD = ED$

$\therefore \quad \Delta ABC \cong \Delta ECD$

$\therefore \quad AB = EC$

Now in ΔACE,

$EC + AC > AE$

or $AB + AC > 2AD \qquad (\because AD = ED \Rightarrow AE = 2AD)$

CHECK YOUR PROGRESS 11.3

1. PQRS is a quadrilateral in which diagonals PR and QS intersect at O. Prove that $PQ + QR + RS + SP > PR + QS$.
2. In triangle ABC, AB = 5.7 cm, BC = 6.2 cm and CA = 4.8 cm. Name the greatest and the smallest angle.
3. In Fig. 11.46, if $\angle CBD > \angle BCE$ then prove that $AB > AC$.

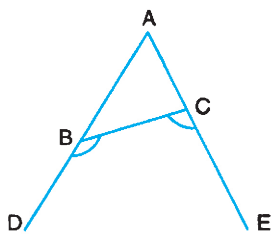

Fig. 11.46

4. In Fig. 11.47, D is any point on the base BC of a ΔABC. If AB > AC then prove that AB > AD.

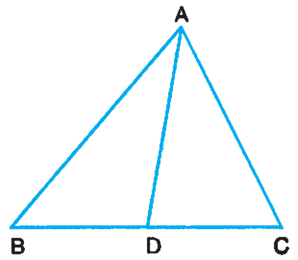

Fig. 11.47

5. Prove that the sum of the three sides of triangle is greater than the sum of its three medians.

(Use Example 11.10)

6. In Fig. 11.48, if AB = AD then prove that BC > CD.

[Hint : ∠ADB = ∠ABD].

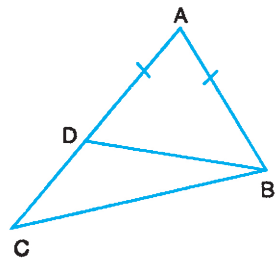

Fig. 11.48

7. In Fig. 11.49, AB is parallel to CD. If ∠A > ∠B then prove that BC > AD.

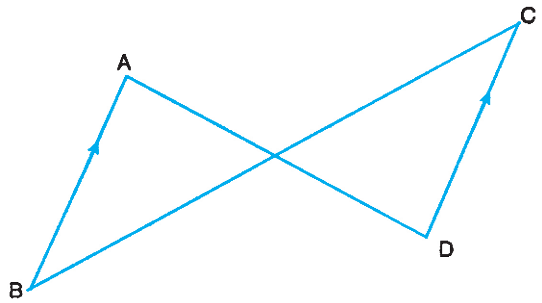

Fig. 11.49

LET US SUM UP

- Figures which have the same shape and same size are called congruent figures.
- Two congruent figures, when placed one over the other completely cover each other. All parts of one figure are equal to the corresponding parts of the other figure.

- To prove that two triangles are congruent we need to know the equality of only three corresponding parts. These corresponding parts must satisfy one of the four criteria.
  (i) SAS (ii) ASA or AAS
  (iii) SSS (iv) RHS
- Angles opposite to equal sides of a triangle are equal.
- Sides opposite to equal angles of a triangle are equal.
- If two sides of a triangle are unequal, then the longer side has the greater angle opposite to it.
- In a triangle, the greater angle has the longer side opposite to it.
- Sum of any two sides of a triangle is greater than the third side.

TERMINAL EXERCISE

1. Two lines AB and CD bisect each other at O. Prove that CA = BD (Fig. 11.50)

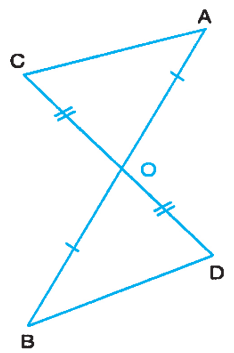

Fig. 11.50

2. In a ΔABC, if the median AD is perpendicular to the base BC then prove that the triangle is an isosceles triangle.
3. In Fig. 11.51, ΔABC and ΔCDE are such that BC = CE and AB = DE. If $\angle B = 60^o$, $\angle ACE = 30^o$ and $\angle D = 90^o$, then prove that the two triangles are congruent.

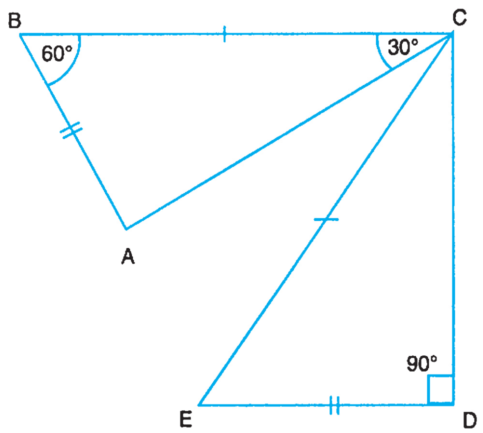

Fig. 11.51

4. In Fig. 11.52 two sides AB and BC and the altitude AD of ΔABC are respectively equal to the sides PQ and QR and the altitude PS, Prove that $\Delta ABC \cong \Delta PQR$.

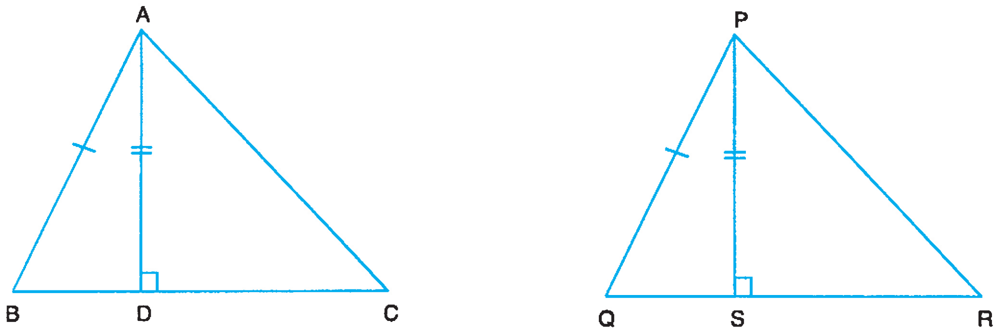

Fig. 11.52

5. In a right triangle, one of the acute angles is 30^o. Prove that the hypotenuse is twice the side opposite to the angle of 30^o.

6. Line segments AB and CD intersect each other at O such that O is the midpoint of AB. If AC is parallel to DB then prove that O is also the mid piont of CD.

7. In Fig. 11.53, AB is the longest side and DC is the shortest side of a quadrilateral ABCD. Prove that $\angle C > \angle A$ and $\angle D > \angle B$. [Hint : Join AC and BD].

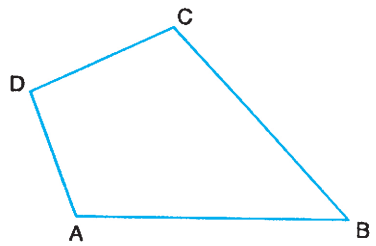

Fig. 11.53

8. ABC is an isosceles triangle in which AB = AC and AD is the altitude from A to the base BC. Prove that BD = DC.

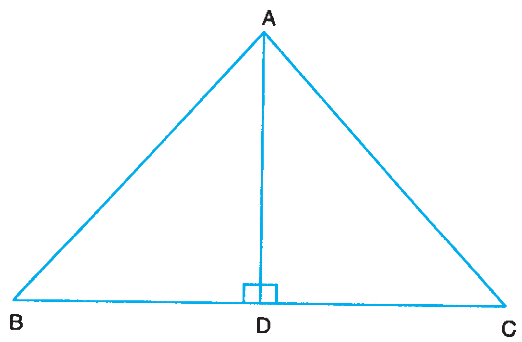

Fig. 11.54

9. Prove that the medians bisecting the equal sides of an isosceles triangle are also equal. [Hint : Show that $\Delta DBC \cong \Delta ECB$]

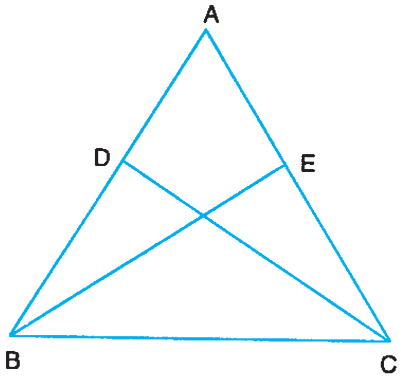

Fig. 11.55

ANSWERS TO CHECK YOUR PROGRESS

11.1

1. (a) 2. (b)

3. (b) 4. (c)

8. $\angle P = \angle C$ $\angle Q = \angle A$ and $\angle R = \angle B$.

11.2

3. $\angle B = \angle C = 65^o$, $\angle A = 50^o$

11.3

2. Greatest angle is A and smallest angle is B.