10

211en10

LINES AND ANGLES

Observe the top of your desk or table. Now move your hand on the top of your table. It gives an idea of a plane. Its edges give an idea of a line, its corner, that of a point and the edges meeting at a corner give an idea of an angle.

OBJECTIVES

After studying this lesson, you will be able to

- *illustrate the concepts of point, line, plane, parallel lines and interesecting lines;*
- *recognise pairs of angles made by a transversal with two or more lines;*
- *verify that when a ray stands on a line, the sum of two angles so formed is* 180^0*;*
- *verify that when two lines intersect, vertically opposite angles are equal;*
- *verify that if a transversal intersects two parallel lines then corresponding angles in each pair are equal;*
- *verify that if a transversal intersects two parallel lines then*

  (a) alternate angles in each pair are equal

  (b) interior angles on the same side of the transversal are supplementary;
- *prove that the sum of angles of a triangle is* 180^0
- *verify that the exterior angle of a triangle is equal to the sum of two interior opposite angles; and*
- *explain the concept of locus and exemplify it through daily life situations.*
- *find the locus of a point equidistent from (a) two given points, (b) two intersecting lines.*
- *solve problems based on starred result and direct numerical problems based on unstarred results given in the curriculum.*

EXPECTED BACKGROUND KNOWLEDGE

- point, line, plane, intersecting lines, rays and angles.
- parralel lines

10.1 POINT, LINE AND ANGLE

In earlier classes, you have studied about a point, a line, a plane and an angle. Let us quickly recall these concepts.

Point : If we press the tip of a pen or pencil on a piece of paper, we get a fine dot, which is called a point.

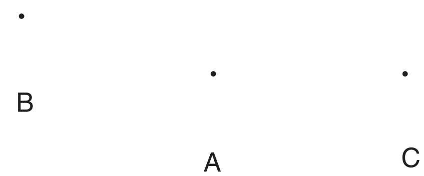


Fig. 10.1

A point is used to show the location and is represented by capital letters A, B, C etc.

10.1.1 Line

Now mark two points A and B on your note book. Join them with the help of a ruler or a scale and extend it on both sides. This gives us a straight line or simply a line.

Fig. 10.2

In geometry, a line is extended infinitely on both sides and is marked with arrows to give this idea. A line is named using any two points on it, viz, AB or by a single small letter *l*, *m* etc. (See fig. 10.3)

Fig. 10.3

The part of the line between two points A and B is called a line segment and will be named AB.

Observe that a line segment is the shortest path between two points A and B. (See Fig. 10.4)

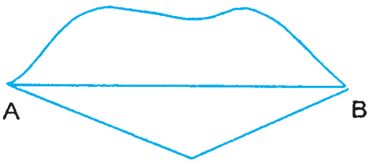


Fig. 10.4

10.1.2 Ray

If we mark a point X and draw a line, starting from it extending infinitely in one direction only, then we get a ray XY.

Fig. 10.5

X is called the initial point of the ray XY.

10.1.3 Plane

If we move our palm on the top of a table, we get an idea of a plane.

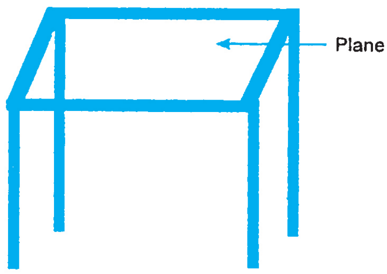


Fig. 10.6

Similarly, floor of a room also gives the idea of part of a plane.

Plane also extends infintely lengthwise and breadthwise.

Mark a point A on a sheet of paper.

How many lines can you draw passing though this point? As many as you wish.

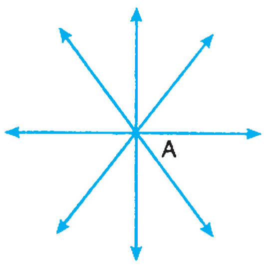


Fig. 10.7

In fact, we can draw an infinite number of lines through a point.

Take another point B, at some distance from A. We can again draw an infinite number of lines passing through B.

Fig. 10.8

Out of these lines, how many pass through both the points A and B? Out of all the lines passing through A, only one passes through B. Thus, only one line passes through both the points A and B. We conclude that **one and only one line can be drawn passing through two given points.**

Now we take three points in plane.

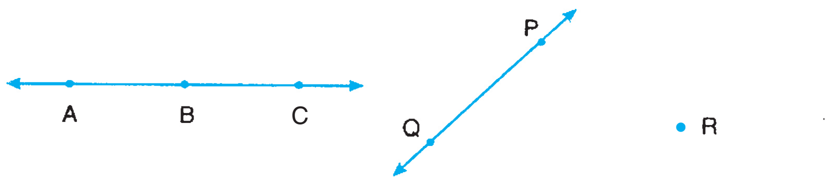


Fig. 10.9

We observe that a line may or may not pass through the three given points.

If a line can pass through three or more points, then these points are said to be **collinear.** For example the points A, B and C in the Fig. 10.9 are collinear points.

If a line **can not** be drawn passing through all three points (or more points), then they are said to be **non-collinear.** For example points P, Q and R, in the Fig. 10.9, are non-collinear points.

Since two points always lie on a line, we talk of collinear points only when their number is three or more.

Let us now take two distinct lines AB and CD in a plane.

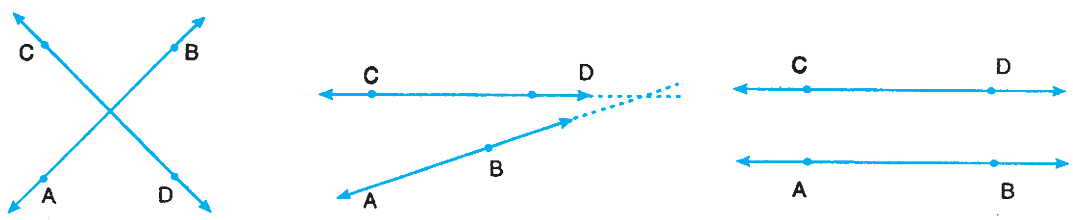


Fig. 10.10

How many points can they have in common? We observe that these lines can have. either (i) one point in common as in Fig. 10.10 (a) and (b). [In such a case they are called intersecting lines] or (ii) no points in common as in Fig. 10.10 (c). In such a case they are called **parrallel lines**.

Now observe three (or more) distinct lines in plane.

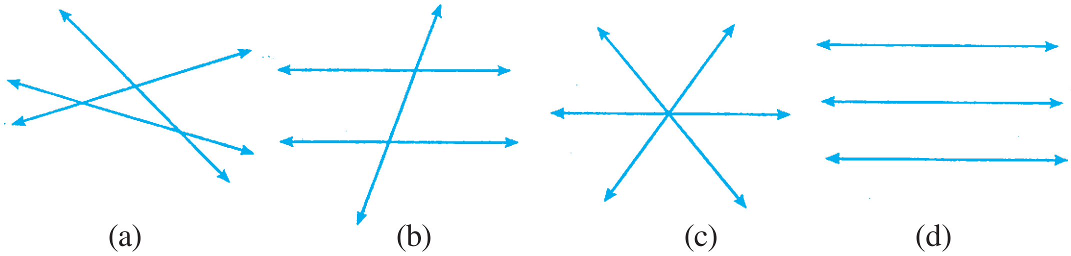


Fig. 10.11

What are the possibilities ?

(i) They may interest in more than one point as in Fig. 10.11 (a) and 10.11 (b).

or (ii) They may intesect in one point only as in Fig. 10.11 (c). In such a case they are called concurrent lines.

or (iii) They may be non intersecting lines parrallel to each other as in Fig. 10.11 |(d).

10.1.4 Angle

Mark a point O and draw two rays OA and OB starting from O. The figure we get is called an angle. Thus, an angle is a figure consisting of two rays starting from a common point.

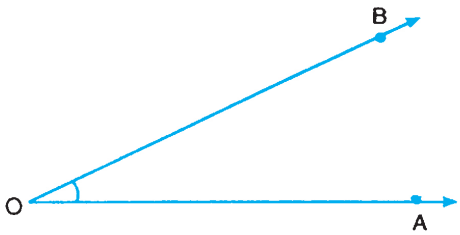


Fig. 10.11(A)

This angle may be named as angle AOB or angle BOA or simply angle O; and is written as $\angle AOB$ or $\angle BOA$ or $\angle O$. [see Fig. 10.11A]

An angle is measured in degrees. If we take any point O and draw two rays starting from it in opposite directions then the measure of this angle is taken to be 180^0 degrees, written as 180^0.

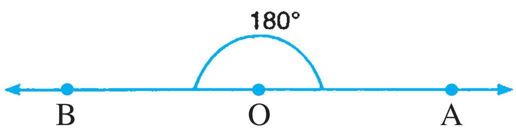


Fig. 10.12

This measure divided into 180 equal parts is called one degree (written as 1^o).

Angle obtained by two opposite rays is called a **straight angle**.

An angle of 90^0 is called a **right angle**, for example $\angle BOA$ or $\angle BOC$ is a right angle in Fig. 10.13.

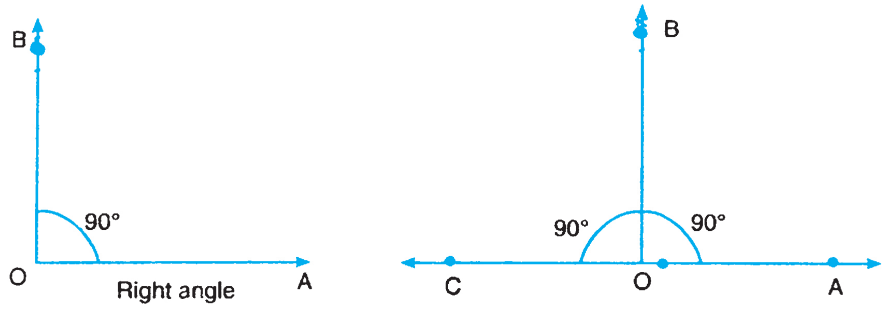


Fig. 10.13

Two lines or rays making a right angle with each other are called **perpendicular lines**. In Fig. 10.13 we can say OA is perpendicular to OB or vice-versa.

An angle less than 90^0 is called an **acute angle**. For example $\angle POQ$ is an acute angle in Fig. 10.14(a).

An angle greater than 90^0 but less than 180^0 is called an **obtuse angle**. For example, $\angle XOY$ is an obtuse angle in Fig. 10.14(b).

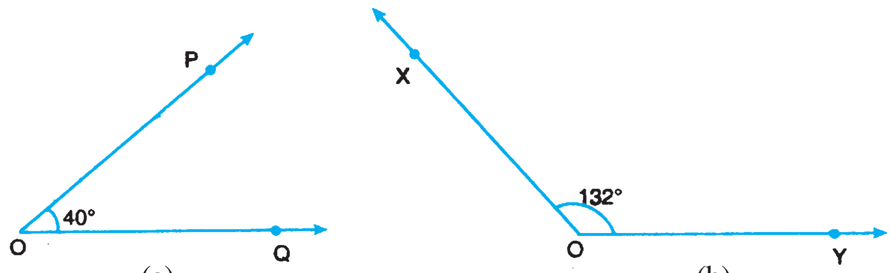


Fig. 10.14

10.2 PAIRS OF ANGLES

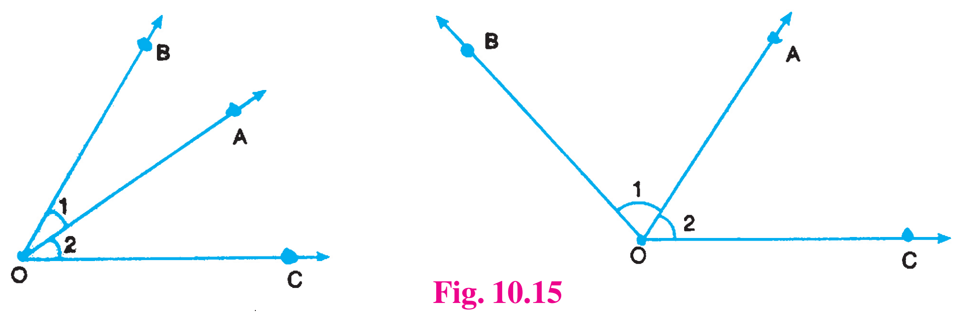


Fig. 10.15

Observe the two angles ∠1 and ∠2 in each of the figures in Fig. 10.15. Each pair has a common vertex O and a common side OA in between OB and OC. Such a pair of angles is called a '**pair of adjacent angles**'.

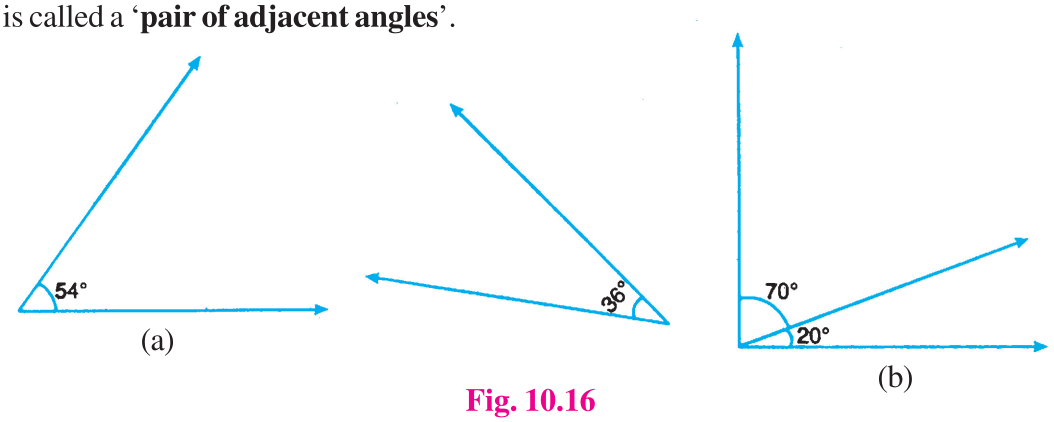


Fig. 10.16

Observe the angles in each pair in Fig. 10.16[(a) and (b)]. They add up to make a total of 90°.

A pair of angles, whose sum is 90°, is called a pair of **complementary angles**. Each angle is called the **complement** of the other.

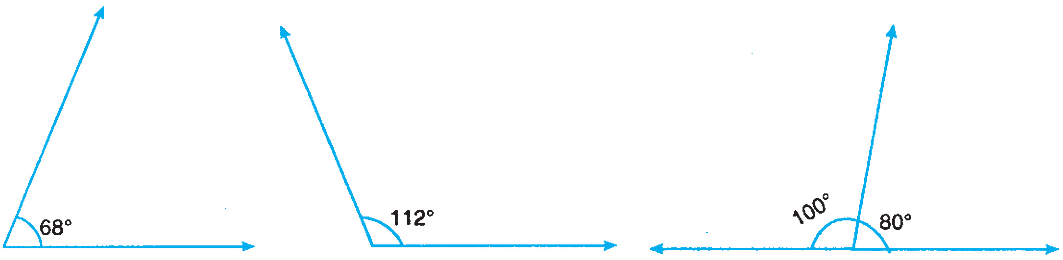


Fig. 10.17

Again observe the angles in each pair in Fig. 10.17[(a) and (b)].

These add up to make a total of 180°.

A pair of angles whose sum is 180°, is called a pair of supplementary angles.

Each such angle is called the **supplement** of the other.

Draw a line AB. From a point C on it draw a ray CD making two angles ∠X and ∠Y.

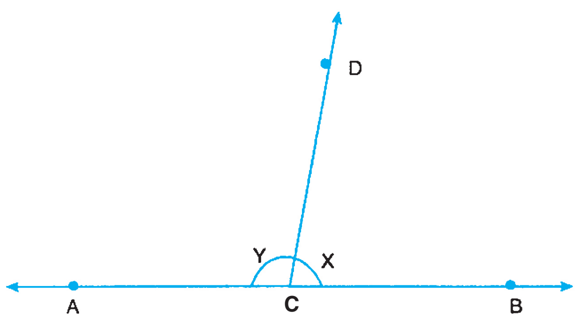


Fig. 10.18

If we measure $\angle X$ and $\angle Y$ and add, we will always find the sum to be 180^0, whatever be the position of the ray CD. We conclude

If a ray stands on a line then the sum of the two adjacent angles so formed is 180^o.

The pair of angles so formed as in Fig. 10.18 is called a **linear pair** of angles.

Note that they also make a pair of supplementary angles.

Draw two intersecting lines AB and CD, intersecting each other at O.

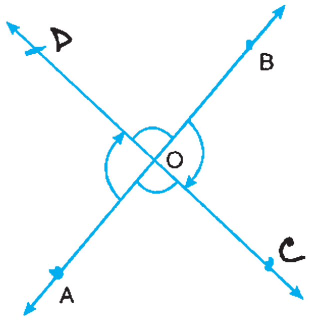


Fig. 10.19

$\angle AOC$ and $\angle DOB$ are angles opposite to each other. These make a pair of **vertically oppposite angles**. Measure them. You will always find that

$\angle AOC = \angle DOB$.

$\angle AOD$ and $\angle BOC$ is another pair of vertically opposite angles. On measuring, you will again find that

$\angle AOD = \angle BOC$

We conclude :

If two lines intersect each other, the pair of vertically opposite angles are equal.

An activity for you.

Attach two strips with a nail or a pin as shown in the figure.

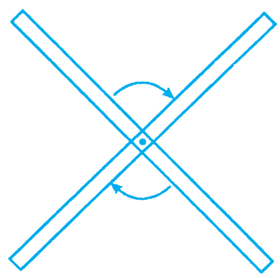


Fig. 10.20

Rotate one of the strips, keeping the other in position and observe that the pairs of vertically opposite angles thus formed are always equal.

A line which intersects two or more lines at distinct points is called a **transversal**. For example line *l* in Fig. 10.21 is a transversal.

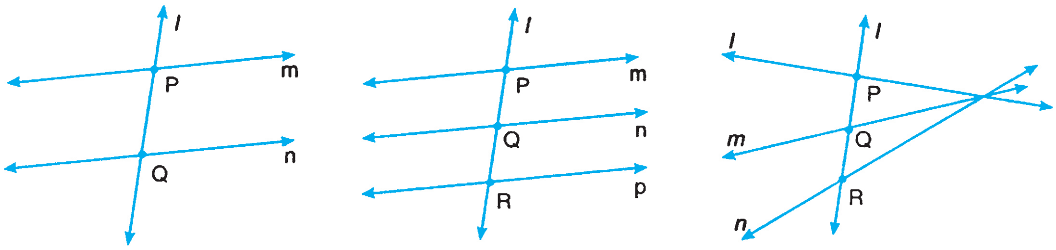


Fig. 10.21

When a transversal intersects two lines, eight angles are formed.

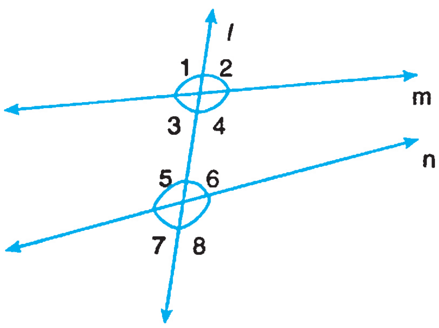


Fig. 10.22

These angles in pairs are very important in the study of properties of parallel lines. Some of the useful pairs are as follows :

(a) $\angle 1$ and $\angle 5$ is a pair of corresponding angles. $\angle 2$ and $\angle 6$, $\angle 3$ and $\angle 7$ and $\angle 4$ and $\angle 8$ are other pairs of corresponding angles.

(b) $\angle 3$ and $\angle 6$ is a pair of alternate angles. $\angle 4$ and $\angle 5$ is another pair of alternate angles.

(c) $\angle 3$ and $\angle 5$ is a pair of interior angles on the same side of the transversal.

$\angle 4$ and $\angle 6$ is another pair of interior angles.

In Fig. 10.22 above, lines m and n are not parallel; as such, there may not exist any relation between the angles of any of the above pairs. However, when lines are parallel, there are some very useful relations in these pairs, which we study in the following:

When a transversal intersects two parallel lines, eight angles are formed, whatever be the position of parallel lines or the transversal.

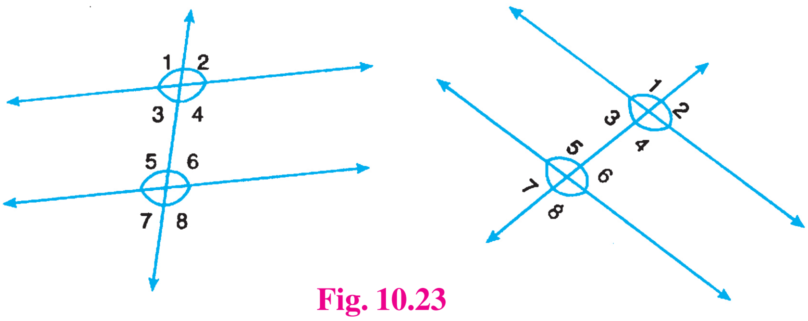


Fig. 10.23

If we measure the angles, we shall alwys find that

$$\angle 1 = \angle 5, \qquad \angle 2 = \angle 6, \qquad \angle 3 = \angle 7 \text{ and } \angle 4 = \angle 8$$

that is, angles in each pair of corresponding angles are equal.

Also $\qquad \angle 3 = \angle 6$ and $\angle 4 = \angle 5$

that is, angles in each pair of alternate angle are equal.

Also, $\qquad \angle 3 + \angle 5 = 180^\circ$ and $\angle 4 + \angle 6 = 180^\circ$.

Hence we conclude :

When a transversal intersects two parallel lines, then angles in

(i) each pair of corresponding angles are equal

(ii) each pair of alternate angles are equal

(iii) each pair of interior angles on the same side of the transversal are supplementary,

You may also verify the truth of these results by drawing a pair of parallel lines (using parallel edges of your scale) and a transversal and measuring angles in each of these pairs.

Converse of each of these results is also true. To verify the truth of the first converse, we draw a line AB and mark two points C and D on it.

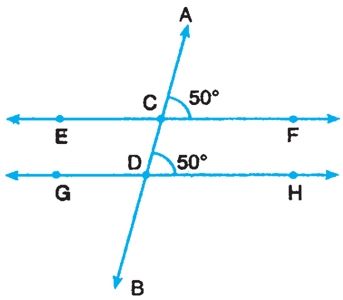


Fig. 10.24

At C and D, we construct two angles ACF and CDH equal to each other, say 50°, as shown in Fig. 10.24. On producing EF and GH on either side, we shall find that they do not intersect each other, that is, they are parallel.

In a similar way, we can verify the truth of the other two converses.

Hence we conclude that

When a transversal inersects two lines in such a way that angles in

(i) any pair of corresponding angles are equal

or (ii) any pair of alternate angles are equal

or (iii) any pair of interior angles on the same side of transversal are supplementary then the two lines are parallel.

Example 10.1 : Choose the correct answwer out of the alternative options in the following multiple choice questions.

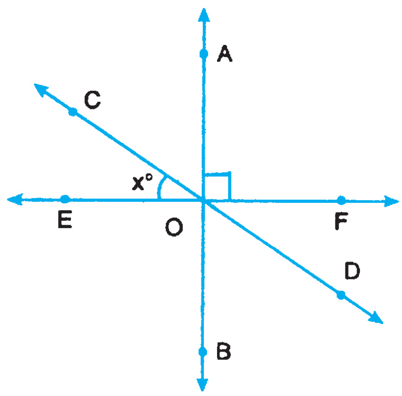


Fig. 10.25

(i) In Fig. 10.25, $\angle FOD$ and $\angle BOD$ are

(A) supplementary angles (B) complementary angles

(C) vertically opposite angles (D) a linear pair of angles **Ans.** (B)

(ii) In Fig. 10.25, $\angle COE$ and $\angle BOE$ are

(A) complementary angles (B) supplementary angles

(C) a linear pair (D) adjacent angles **Ans.** (D)

(iii) In Fig. 10.25, $\angle BOD$ is equal to

(A) x° (B) $(90 + x)^\circ$

(C) $(90 - x)^\circ$ (D) $(180 - x)^\circ$ **Ans** (C)

(iv) An angle is 4 times its supplement; the angle is

(A) 39° (B) 72°

(C) 108° (D) 144° **Ans** (D)

(v) What value of x will make ACB a straight angle in Fig. 10.26

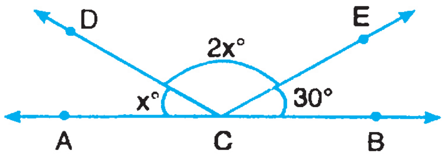


Fig. 10.26

(A) 30° (B) 40°

(C) 50° (D) 60° **Ans** (C)

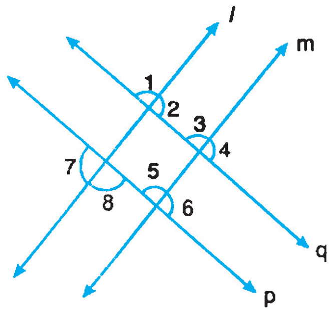


Fig. 10.27

In the above figure, l is parallel to m and p is parallel to q.

(vi) $\angle 3$ and $\angle 5$ form a pair of

(A) Alternate angles (B) interior angles

(C) vertically opposite (D) corresponding angles **Ans** (D)

(vii) In Fig. 10.27, if $\angle 1 = 80^o$, then $\angle 6$ is equal to

(A) 80° (B) 90°

(C) 100° (D) 110° **Ans** (C)

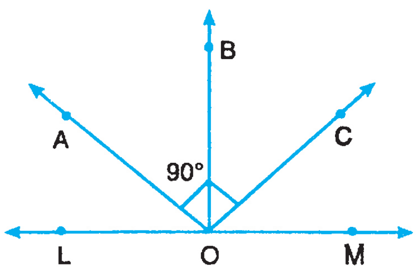


Fig. 10.28

(viii) In Fig. 10.28, OA bisects $\angle LOB$, OC bisects $\angle MOB$ and $\angle AOC = 90^0$. Show that the points L, O and M are collinear.

Solution : $\angle BOL = 2\ \angle BOA$ ...(i)

and $\angle BOM = 2\ \angle BOC$ ...(ii)

Adding (i) and (ii), $\angle BOL + \angle BOM = 2\ \angle BOA + 2\angle BOC$

$\therefore\ \angle LOM = 2[\angle BOA + \angle BOC]$

$= 2 \times 90^o$

$= 180^o$ = a straight angle

$\therefore$ L, O and M are collinear.

CHECK YOUR PROGRESS 10.1.

1. Choose the correct answer out of the given alternatives in the following multiple choice questions :

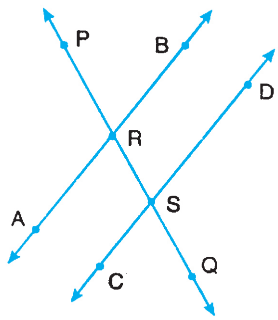


Fig. 10.29

In Fig. 10.29, AB || CD and PQ intersects them at R and S respectively.

(i) $\angle ARS$ and $\angle BRS$ form

(A) a pair of alternate angles

(B) a linear pair

(C) a pair of corresponding angles

(D) a pair of vertically opposite angles

(ii) $\angle ARS$ and $\angle RSD$ form a pair of

(A) Alternate angles (B) Vertically opposite angles

(C) Corresponding angles (D) Interior angles

(iii) If $\angle PRB = 60^o$, then $\angle QSC$ is

(A) 120^o (B) 60^o

(C) 30° (D) 90°

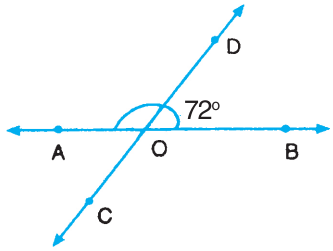


Fig. 10.30

(iv) In Fig. 10.30 above, AB and CD intersect at O. ∠COB is equal to

(A) 36° (B) 72°

(C) 108° (D) 144°

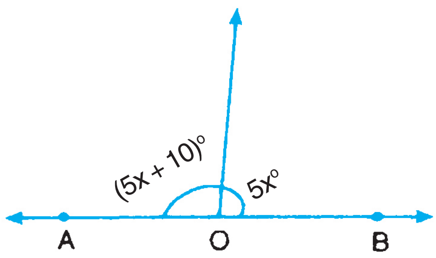


Fig. 10.31

2. In Fig. 10.31 above, AB is a straight line. Find x

3. In Fig. 10.32 below, *l* is parallel to m. Find angles 1 to 7.

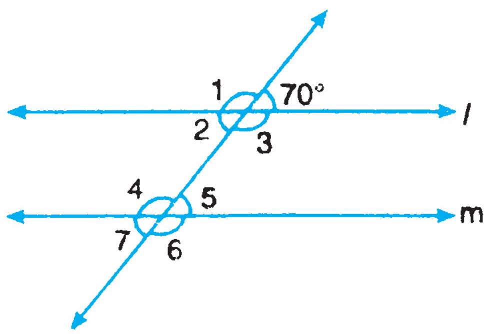


Fig. 10.32

10.3 TRIANGLE, ITS TYPES AND PROPERTIES

Triangle is the simplest polygon of all the closed figures formed in a plane by three line segments.

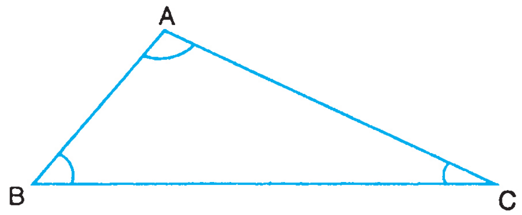


Fig. 10.33

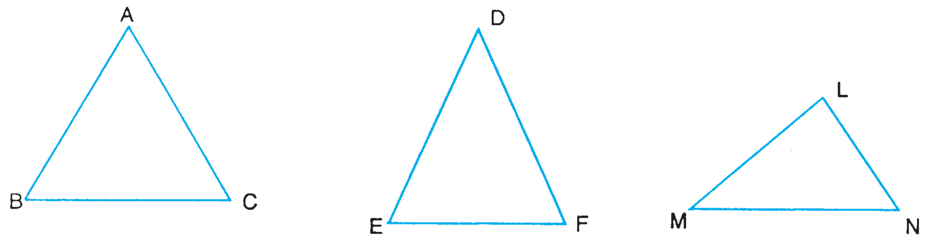
It is a closed figure formed by three line segments having six elements, namely three **angles** (i) ∠ABC or ∠B (ii) ∠ACB or ∠C (iii) ∠CAB or ∠A and three **sides** : (iv) AB (v) BC (vi) CA

It is named as Δ ABC or Δ BAC or Δ CBA and read as triangle ABC or triangle BAC or triangle CBA.

10.3.1 Types of Triangles

Triangles can be classified into different types in two ways.

(a) On the basis of sides



(i) (ii) (iii)

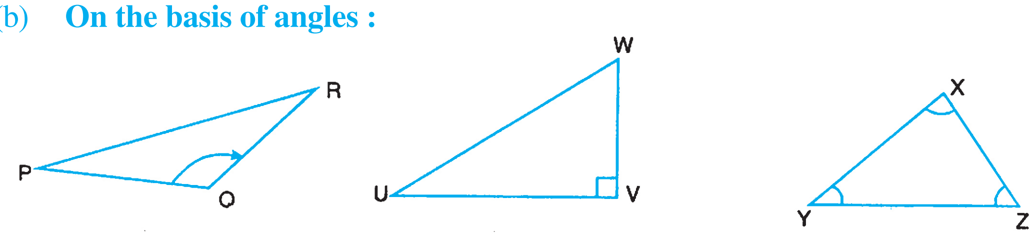
Fig. 10.34

(i) **Equilateral triangle :** a triangle in which all the three sides are equal is called an equilateral trangle. [Δ ABC in Fig. 10.34(i)]

(ii) **Isosceles triangle :** A triangle in which two sides are equal is called an isosceles triangle. [ΔDEF in Fig. 10.34(ii)]

(iii) **Scalene triangle :** A triangle in which all sides are of different lengths, is called a sclene triangle [Δ LMN in Fig. 10.34(iii)]

(b) **On the basis of angles :**



(i) (ii) (iii)

Fig. 10.35

(i) **Obtuse angled triangle :** A triangle in which one of the angles is an obtuse angle is called an **obtuse angled triangle** or simply obtuse triangle [Δ PQR is Fig. 10.35(i)]

(ii) **Right angled triangle :** A triangle in which one of the angles is a right angle is called a **right angled triangle** or right triangle. [Δ UVW in Fig. 10.35(ii)]

(iii) **Acute angled triangle :** A triangle in which all the three angles are acute is called an **acute angled triangle** or acute triangle [Δ XYZ in Fig. 10.35(iii)

Now we shall study some important properties of angles of a triangle.

10.3.2 Angle Sum Property of a Triangle

We draw two triangles and measure their angles.

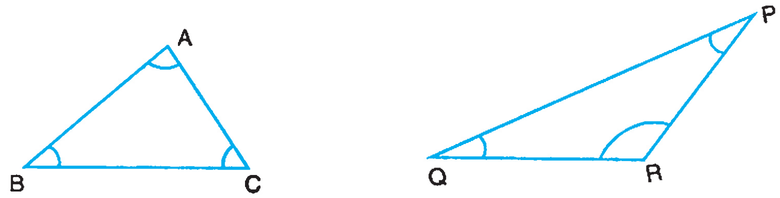


Fig. 10.36

In Fig. 10.36 (a), $\angle A = 80^o$, $\angle B = 40^o$ and $\angle C = 60^o$

$\therefore \quad \angle A + \angle B + \angle C = 80^o + 40^o + 60^o = 180^o$

In Fig. 10.36(b), $\angle P = 30^o$, $\angle Q = 40^o$, $\angle R = 110^o$

$\therefore \quad \angle P + \angle Q + \angle R = 30^o + 40^o + 110^o = 180^o$

What do you observe? Sum of the angles of triangle in each case in 180^0.

We will prove this result in a logical way naming it as a theorem.

Theorem : The sum of the three angles of triangle is 180º.

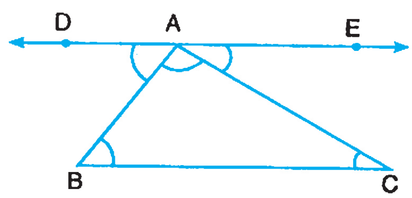


Fig. 10.37

Given : A triangle ABC

To Prove : $\angle A + \angle B + \angle C = 180^0$

Construction : Through A, draw a line DE parallel to BC.

Proof : Since DE is parallel to BC and AB is a transversal.

$\therefore \quad \angle B = \angle DAB$ (Pair of alternate angles)

Similarly $\angle C = \angle EAC$ (Pair of alternate angles)

$\therefore \angle B + \angle C = \angle DAB + \angle EAC$ ...(1)

Now adding $\angle A$ to both sides of (1)

$$\angle A + \angle B + \angle C = \angle A + \angle DAB + \angle EAC$$

$$= 180^o \quad \text{(Angles making a straight angle)}$$

10.3.3 Exterior Angles of a Triangle

Let us produce the side BC of ΔABC to a point D.

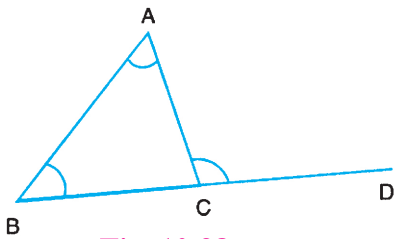


Fig. 10.38

In Fig. 10.39, observe that there are six exterior angles of the ΔABC, namely $\angle 1$, $\angle 2$, $\angle 3$, $\angle 4$, $\angle 5$ and $\angle 6$.

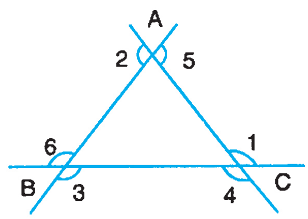


Fig. 10.39

In Fig. 10.38, $\angle ACD$ so obtained is called an exterior angle of the ΔABC. Thus,

The angle formed by a side of the triangle produced and another side of the triangle is called an exterior angle of the triangle.

Corresponding to an exterior angle of a triangle, there are two interior opposite angles.

Interior opposite angles are the angles of the triangle not forming a linear pair with the given exterior angle.

For example in Fig. 10.38, $\angle A$ and $\angle B$ are the two interior opposite angles corresponding to the exterior angle ACD of ΔABC. We measure these angles.

$$\angle A = 60^o$$

$$\angle B = 50^o$$

and $\angle ACD = 110^o$

We observe that $\angle ACD = \angle A + \angle B$.

This observation is true in general.

Thus, we may conclude :

An exterior angle of a triangle is equal to the sum of the two interior opposite angles.

Examples 10.3 : Choose the correct answer out of the given alternatives in the following multiple choice questions:

(i) Which of the following can be the angles of a triangle?

(A) 65^o, 45^o and 80^o (B) 90^o, 30^o and 61^o

(C) 60^o, 60^o and 59^o (D) 60^o, 60^o and 60^o. **Ans** (D)

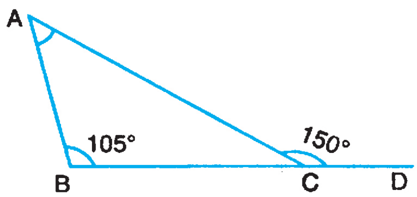


Fig. 10.40

(ii) In Fig. 10.40 $\angle A$ is equal to

(A) 30^o (B) 35^o

(C) 45^o (D) 75^o **Ans** (C)

(iii) In a triangle, one angle is twice the other and the third angle is 60^0. Then the largest angle is

(A) 60^o (B) 80^o

(C) 100^o (D) 120^o **Ans** (B)

Example 10.4:

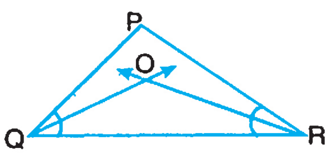


Fig. 10.41

In Fig. 10.41, bisctors of $\angle PQR$ and $\angle PRQ$ intersect each other at O. Prove that $\angle QOR = 90^o + \frac{1}{2}\angle P$.

Solution : $\angle QOR = 180^o - \frac{1}{2}[\angle PQR + \angle PRQ)]$

$$= 180^o - \frac{1}{2}(\angle PQR + \angle PRQ)$$

$$= 180^o - \frac{1}{2}(180^o \quad - \angle P)$$

$$= 180^o - 90^o + \frac{1}{2}\angle P = 90^o + \frac{1}{2}\angle P$$

CHECK YOUR PROGRESS 10.2

1. Choose the correct answer out of given alternatives in the following multiple choice questions:

(i) A triangle can have

(A) Two right angles (B) Two obtuse angles

(C) At the most two acute angles (D) All three acute angles

(ii) In a right triangle, one exterior angles is 120^0, The smallest angle of the triangles is

(A) 20^o (B) 30^0

(C) 40^o (D) 60^0

(iii)

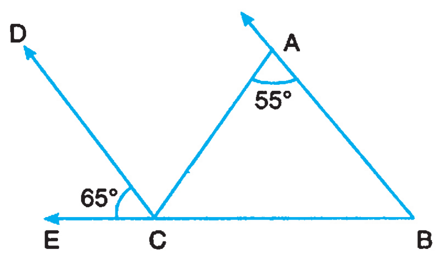


Fig. 10.42

In Fig. 10.42, CD is parallel to BA. $\angle ACB$ is equal to

(A) 55^o (B) 60^o

(C) 65^o (D) 70^o

2. The angles of a triangle are in the ratio 2 : 3 : 5, find the three angles.

3. Prove that the sum of the four angles of a quadrilateral is 360^o.

4. In Fig. 10.43, ABCD is a trapezium such that AB||DC. Find ∠D and ∠C and verify that sum of the four angles is 360°.

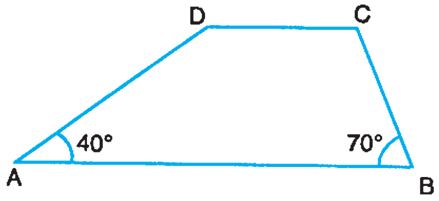


Fig. 10.43

5. Prove that if one angle of a triangle is equal to the sum of the other two angles, then it is a right triangle.

6. In Fig. 10.44, ABC is triangle such that ∠ABC = ∠ACB. Find the angles of the triangle.

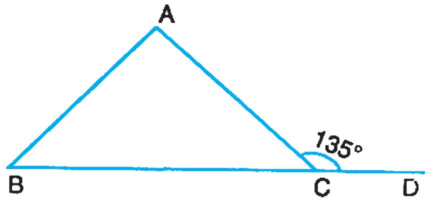


Fig. 10.44

10.4 LOCUS

During the game of cricket, when a player hits the ball, it describes a path, before being caught or touching the ground.

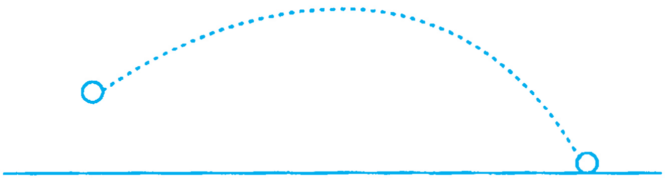


Fig. 10.44

The path described is called Locus.

A figure in geometry is a result of the path traced by a point (or a very small particle) moving under certain conditions.

For example:

(1) Given two parallel lines *l* and m, also a point P between them equidistant from both the lines.

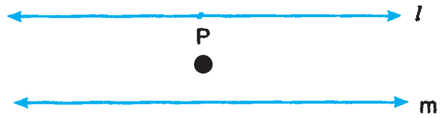


Fig. 10.45

If the particle moves so that it is equidistant from both the lines, what will be its path?

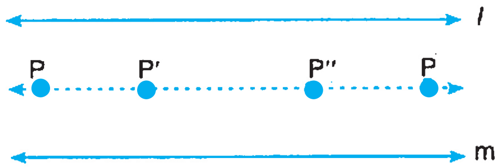


Fig. 10.46

The path traced by P will be a line parallel to both the lines and exactly in the middle of them as in Fig. 10.46.

(2) Given a fixed point O and a point P at a fixed distance *d*.

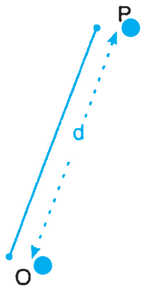


Fig. 10.47

If the point P moves in a plane so that it is always at a constant distance *d* from the fixed point O, what will be its path?

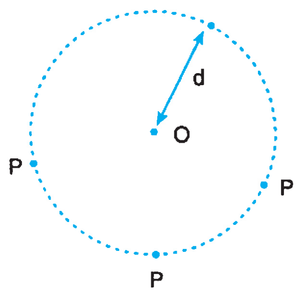


Fig. 10.48

The path of the moving point P will be a circle as shown in Fig. 10.48.

(3) Place a small piece of chalk stick or a pebble on top of a table. Strike it hard with a pencil or a stick so that it leaves the table with a certain speed and observe its path after it leaves the table.

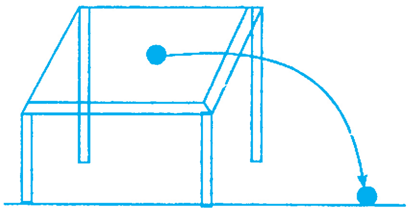


Fig. 10.49

The path traced by the pebble will be a curve (part of what is known as a parabola) as shown in Fig. 10.49.

Thus, locus of a point moving under certain conditions is the path or the geometrical figure, every point of which satisfies the given conditon(s).

10.4.1 Locus of a point equidistant from two given points

Let A and B be the two given points.

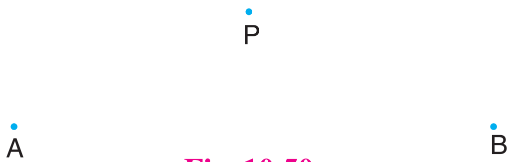


Fig. 10.50

We have to find the locus of a point P such that PA = PB.

Joint AB. Mark the mind point of AB as M. Clearly, M is a point which is equidistant from A and B. Mark another point P using compasses such that PA = PB. Join PM and extend it on both sides. Using a pair of divider or a scale, it can easily be verified that every point on PM is equidistant from the points A and B. Also, if we take any other point Q not lying on line PM, then QA $\neq$ QB.

Also $\angle AMP = \angle BMP = 90^o$

That is, PM is the perpendicular bisector of AB.

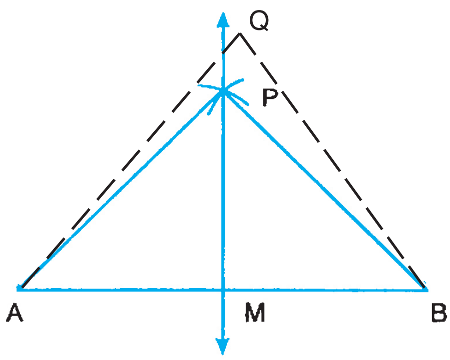


Fig. 10.51

Thus, we may conclude the following:

The locus of a point equidistant from two given poitns is the perpendicular bisector of the line segment joining the two points.

Activity for you :

Mark two points A and B on a sheet of paper and join them. Fold the paper along mid-point of AB so that A coincides with B. Make a crease along the line of fold. This crease is a straight line. This is the locus of the point equidistant from the given points A and B. It can be easily checked that very point on it is equidistant from A and B.

10.4.2 Locus of a point equidistant from two lines intersecting at O

Let AB and CD be two given lines intersecting at O.

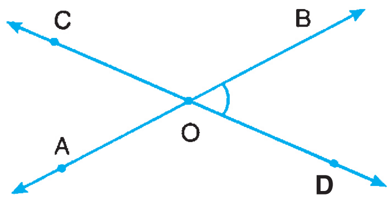


Fig. 10.52

We have to find the locus of a point P which is equidistant from both AB and CD.

Draw bisectors of ∠BOD and ∠BOC.

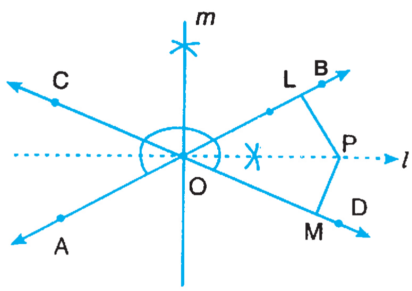


Fig. 10.53

If we take any point P on any bisector *l* or m, we will find perpendicular distances PL and PM of P from the lines AB and CD are equal.

that is, PL = PM

If we take any other point, say Q, not lying on any bisector *l* or m, then QL will not be equal to QM.

Thus, we may conclude :

The locus of a point equidistant from two intersecting lines is the pair of lines, bisecting the angles formed by the given lines.

Activity for you :

Draw two lines AB and CD intersecting at O, on a sheet of paper. Fold the paper through O so that AO falls on CO and OD falls on OB and mark the crease along the fold. Take a piont P on this crease which is the bisector of $\angle$BOD and check using a set square that

PL = PM

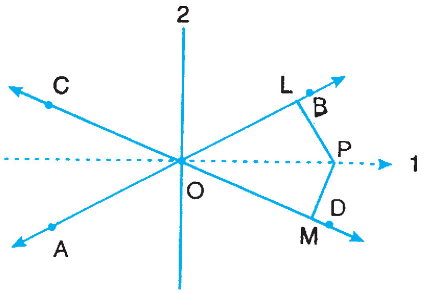


Fig. 10.54

In a similar way find the other bisector by folding again and getting crease 2. Any point on this crease 2 is also equidistant from both the lines.

Example 10.5 : Find the locus of the centre of a circle passing through two given points.

Solution : Let the two given points be A and B. We have to find the position or positions of centre O of a circle passing through A and B.

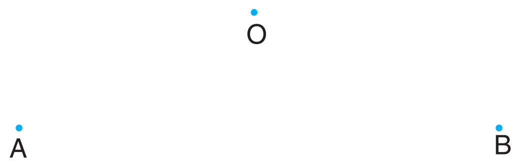


Fig. 10.55

Point O must be equidistant from both the points A and B. As we have already learnt, the locus of the point O will be the perpendicular bisector of AB.

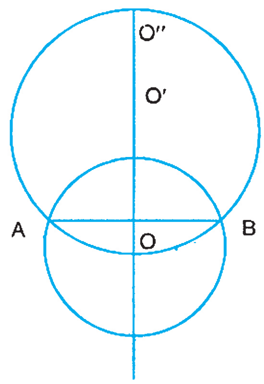


Fig. 10.56

CHECK YOU PROGRESS 10.3

1. Find the locus of the centre of a circle passing through three given points A, B and C which are non-collinear.

2. There are two villages certain distance apart. A well is to be dug so that it is equidistant from the two villages such that its distance from each village is not more than the distance between the two villages. Representing the villages by points A and B and the well by point P. show in a diagram the locus of the point P.

3. Two straight roads AB and CD are intersecting at a point O. An observation post is to be constructred at a distance of 1 km from O and equidistant from the roads AB and CD. Show in a diagram the possible locations of the post.

4. Find the locus of a point which is always at a distance 5 cm from a given line AB.

LET US SUM UP

- A line extends to inifinity on both sides and a line segment is only a part of it between two points.
- Two distinct lines in a plane may either be intersecting or parallel.
- If three or more lines intersect in one point only then they are called cocurrent lines.
- Two rays starting from a common point form an angle.
- A pair of angles, whose sum is 90^0 is called a pair of complementary angles.
- A pair of angles whose sum is 180^0 is called a pair of supplementary angles.
- If a ray stands on a line then the sum of the two adjacent angles, so formed is 180^0
- If two lines intersect each other the pairs of vertically opposite angles are equal
- When a transversal intersects two parallel lines, then

  (i) corresponding angles in a pair are equal.

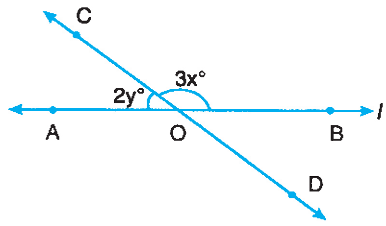
  (ii) alternate angles are equal.

  (iii) interior angles on the same side of the transversal are supplementary.
- The sum of the angles of a triangle is 180^0
- An exterior angle of a triangle to equal to the sum of the two interior opposite angles
- Locus of a point equidistant from two given points is the perpendicular bisector of the line segment joing the points.

- The locus of a point equidistant from the intersecting lines is the pair of lines, bisecting the angle formed by the given lines.

TERMINAL EXERCISE

1. In Fig. 10.57, if x = 42, then determine (a) y (b) $\angle AOD$



2.

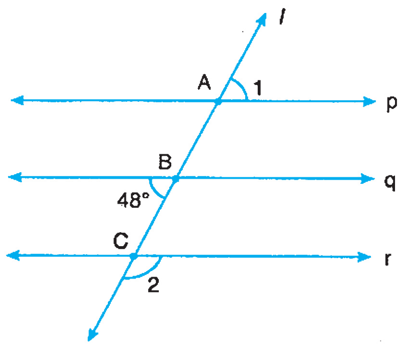


Fig. 10.58

In the above figure p, q and r are parallel lines intersected by a transversal l at A, B and C respectively. Find $\angle 1$ and $\angle 2$.

3. The sum of two angles of a triangle is equal to its third angle. Find the third angle. What type of triangle is it?

4.

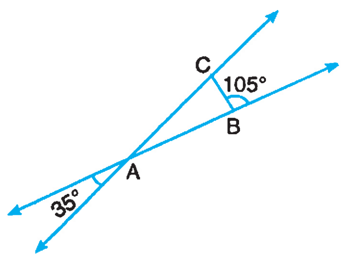


Fig. 10.59

In Fig. 10.59, sides of Δ ABC have been produced as shown. Find the angles of the triangle.

5.

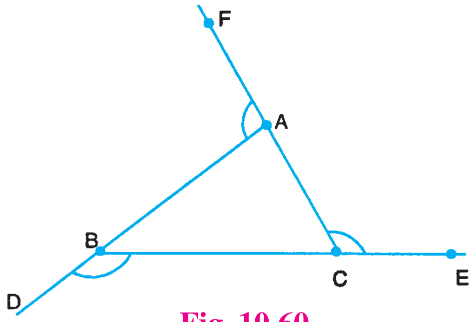


Fig. 10.60

In Fig. 10.60, sides AB, BC and CA of the triangle ABC have been produced as shown. Show that the sum of the exterior angles so formed is 360º.

6.

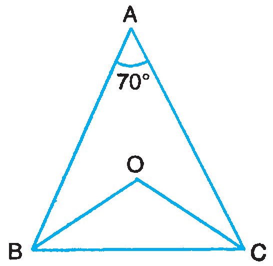


Fig. 10.61

In Fig. 10.61 ABC is a triangle in which bisectors of $\angle B$ and $\angle C$ meet at O. Show that $\angle BOC = 125^o$.

7.

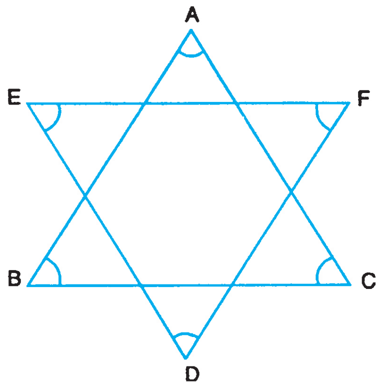


Fig. 10.62

In Fig. 10.62 above, find the sum of the angles, $\angle A$, $\angle F$, $\angle C$, $\angle D$, $\angle B$ and $\angle E$.

8.

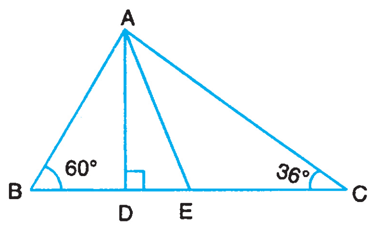


Fig. 10.63

In Fig. 10.63 in Δ ABC, AD is perpendicular to BC and AE is bisector of $\angle$BAC. Find $\angle$DAE,

9.

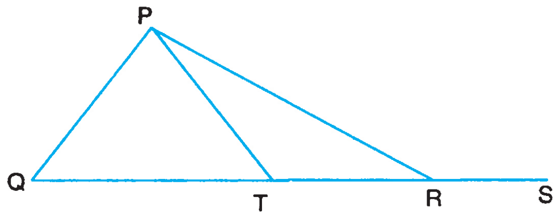


Fig. 10.64

In Fig. 10.64 above, in Δ PQR, PT is bisector of $\angle$P and QR is produced to S. Show that $\angle$PQR + $\angle$PRS = 2 $\angle$PTR.

10. Prove that the sum of the (interior) angles of a pentagon is 540^0.

11. Find the locus of a point equidistant from two parallel lines *l* and m at a distance of 5 cm from each other.

12. Find the locus of a point equidistant from points A and B and also equidistant from rays AB and AC of Fig. 10.65.

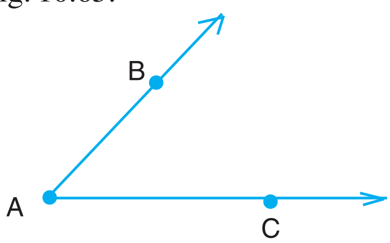


Fig. 10.65

ANSWERS TO CHECK YOUR PROGRESS

10.1

1. (i) (B) (ii) (A) (iii) (B) (iv) (C)

2. $x = 17^0$.

3. $\angle 1 = \angle 3 = \angle 4 = \angle 6 = 110^o$

and $\angle 2 = \angle 5 = \angle 7 = 70^o$.

10.2

1. (i) (D) (ii) (B) (iii) (B)

2. 36^o, 54^o and 90^o 4. $\angle D = 140^o$ and $\angle C = 110^o$

6. $\angle ABC = 45^o$, $\angle ACB = 45^o$ and $\angle A = 90^o$

10.3

1. Only a point, which is the point of intersection of perpendicular bisectors of AB and BC.
2. Let the villages be A and B, then locus will be the line segment PQ, perpendicular bisector of AB such that

   AP = BP = QA = QB = AB

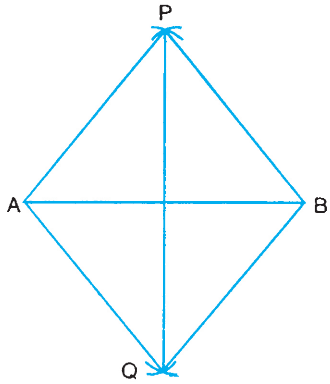


Fig. 10.65

3. Possible locations will be four points two points P and Q on the bisector of ∠AOC and two points R and S on the bisector of ∠BOC.

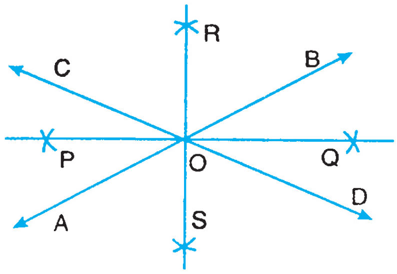


Fig. 10.66

4. Two on either side of AB and lines parallel to AB at a distance of 5 cm from AB.

ANSWERS TO TERMINAL EXERCISE

1. (a) y = 27 (b) = 126°   2. ∠1 = 48° and ∠2 = 132°
3. Third angle = 90°, Right triangle   4. ∠A = 35°, ∠B = 75° ∠C = 70°
7. 360°   8. 12°
11. A line parallel to locus *l* and m at a distance of 2.5 cm from each.
12. Point of intersection of the perpendicular bisector of AB and bisector of ∠BAC.