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NUMBER SYSTEMS

From time immemorial human beings have been trying to have a count of their belongings- goods, ornaments, jewels, animals, trees, sheeps/goats, etc. by using various techniques

- putting scratches on the ground/stones

- by storing stones - one for each commodity kept/taken out.

This was the way of having a count of their belongings without having any knowledge of counting.

One of the greatest inventions in the history of civilization is the creation of numbers. You can imagine the confusion when there were no answers to questions of the type "How many?", "How much?" and the like in the absence of the knowledge of numbers. The invention of number system including zero and the rules for combining them helped people to reply questions of the type:

(i) How many apples are there in the basket?

(ii) How many speakers have been invited for addressing the meeting?

(iii) What is the number of toys on the table?

(iv) How many bags of wheat have been the yield from the field?

The answers to all these situations and many more involve the knowledge of numbers and operations on them. This points out to the need of study of number system and its extensions in the curriculum. In this lesson, we will present a brief review of natural numbers, whole numbers and integers. We shall then introduce you about rational and irrational numbers in detail. We shall end the lesson after discussing about real numbers.

OBJECTIVES

After studying this lesson, you will be able to

- *illustrate the extension of system of numbers from natural numbers to real (rationals and irrational) numbers*

- *identify different types of numbers;*
- *express an integer as a rational number;*
- *express a rational number as a terminating or non-terminating repeating decimal, and vice-versa;*
- *find rational numbers between any two rationals;*
- *represent a rational number on the number line;*
- *cites examples of irrational numbers;*
- *represent* $\sqrt{2}, \sqrt{3}, \sqrt{5}$ *on the number line;*
- *find irrational numbers betwen any two given numbers;*
- *round off rational and irrational numbers to a given number of decimal places;*
- *perform the four fundamental operations of addition, subtraction, multiplication and division on real numbers.*

1.1 EXPECTED BACKGROUND KNOWLEDGE

Basic knowledge about counting numbers and their use in day-to-day life.

1.2 RECALL OF NATURAL NUMBERS, WHOLE NUMBERS AND INTEGERS

1.2.1 Natural Numbers

Recall that the counting numbers 1, 2, 3, ... constitute the system of natural numbers. These are the numbers which we use in our day-to-day life.

Recall that there is no greatest natural number, for if 1 is added to any natural number, we get the next higher natural number, called its successor.

We have also studied about four-fundamental operations on natural numbers. For, example,

$4 + 2 = 6$, again a natural number;

$6 + 21 = 27$, again a natural number;

$22 - 6 = 16$, again a natural number, but

$2 - 6$ is not defined in natural numbers.

Similarly, $4 \times 3 = 12$, again a natural number

$12 \times 3 = 36$, again a natural number

$\frac{12}{2} = 6$ is a natural number but $\frac{6}{4}$ is not defined in natural numbers. Thus, we can say that

i) a) addition and multiplication of natural numbers again yield a natural number but

b) subtraction and division of two natural numbers may or may not yield a natural number

ii) The natural numbers can be represented on a number line as shown below.

iii) Two natural numbers can be added and multiplied in any order and the result obtained is always same. This does not hold for subtraction and division of natural numbers.

1.2.2 Whole Numbers

(i) When a natural number is subtracted from itself we can not say what is the left out number. To remove this difficulty, the natural numbers were extended by the number zero (0), to get what is called the system of whole numbers

Thus, the whole numbers are

0, 1, 2, 3, ...

Again, like before, there is no greatest whole number.

(ii) The number 0 has the following properties:

$$a + 0 = a = 0 + a$$

$a - 0 = a$ but $(0 - a)$ is not defined in whole numbers

$$a \times 0 = 0 = 0 \times a$$

Division by zero (0) is not defined.

(iii) Four fundamental operations can be performed on whole numbers also as in the case of natural numbers (with restrictions for subtraction and division).

(iv) Whole numbers can also be represented on the number line as follows:

1.2.3 Integers

While dealing with natural numbers and whole numbers we found that it is not always possible to subtract a number from another.

For example, (2 – 3), (3 – 7), (9 – 20) etc. are all not possible in the system of natural numbers and whole numbers. Thus, it needed another extension of numbers which allow such subtractions.

Thus, we extend whole numbers by such numbers as –1 (called negative 1), – 2 (negative 2) and so on such that

$$1 + (-1) = 0,\ 2 + (-2) = 0,\ 3 + (-3) = 0\ldots,\ 99 + (-99) = 0,\ \ldots$$

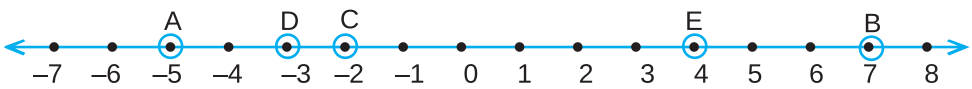
Thus, we have extended the whole numbers to another system of numbers, called integers. The integers therefore are

$$\ldots,\ -7,\ -6,\ -5,\ -4,\ -3,\ -2,\ -1,\ 0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ \ldots$$

1.2.4 Representing Integers on the Number Line

We extend the number line used for representing whole numbers to the left of zero and mark points – 1, – 2, – 3, – 4, ... such that 1 and – 1, 2 and – 2, 3 and – 3 are equidistant from zero and are in opposite directions of zero. Thus, we have the integer number line as follows:

We can now easily represent integers on the number line. For example, let us represent – 5, 7, – 2, – 3, 4 on the number line. In the figure, the points A, B, C, D and E respectively represent – 5, 7, – 2, – 3 and 4.



We note here that if an integer a > b, then 'a' will always be to the right of 'b', otherwise vise-versa.

For example, in the above figure 7 > 4, therefore B lies to the right of E. Similarly, – 2 > – 5, therefore C (– 2) lies to the right of A (–5).

Conversely, as 4 < 7, therefore 4 lies to the left of 7 which is shown in the figure as E is to the left of B

∴ For finding the greater (or smaller) of the two integers a and b, we follow the following rule:

i) a > b, if a is to the right of b

ii) a < b, if a is to the left of b

Example 1.1: Identify natural numbers, whole numbers and integers from the following:-

15, 22, – 6, 7, – 13, 0, 12, – 12, 13, – 31

Solution: Natural numbers are: 7, 12, 13, 15 and 22

whole numbers are: 0, 7, 12, 13, 15 and 22

Integers are: – 31, – 13, – 12, – 6, 0, 7, 12, 13, 15 and 22

Example 1.2: From the following, identify those which are (i) not natural numbers (ii) not whole numbers

– 17, 15, 23, – 6, – 4, 0, 16, 18, 22, 31

Solution: i) – 17, – 6, – 4 and 0 are not natural numbers

ii) – 17, – 6, – 4 are not whole numbers

Note: From the above examples, we can say that

i) all natural numbers are whole numbers and integers also but the vice-versa is not true

ii) all whole numbers are integers also

You have studied four fundamental operations on integers in earlier classes. Without repeating them here, we will take some examples and illustrate them here

Example 1.3: Simplify the following and state whether the result is an integer or not

$12 \times 4,\ 7 \div 3,\ 18 \div 3,\ 36 \div 7,\ 14 \times 2,\ 18 \div 36,\ 13 \times (-3)$

Solution: $12 \times 4 = 48$; it is an integer

$7 \div 3 = \frac{7}{3}$; It is not an integer

$18 \div 3 = 6$; It is an integer

$36 \div 7 = \frac{36}{7}$; It is not an integer.

$14 \times 2 = 28$, It is an integer

$18 \div 36 = \frac{18}{36}$; It is not an integer

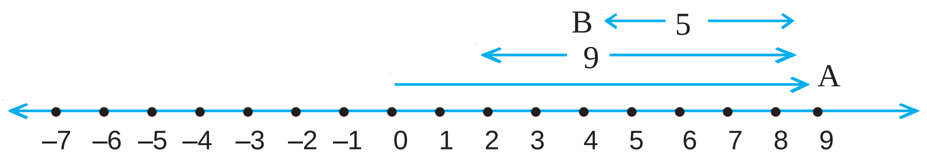
$13 \times (-3) = -39$; It is an integer

Example 1.4: Using number line, add the following integers:

(i) $9 + (-5)$ (ii) $(-3) + (-7)$

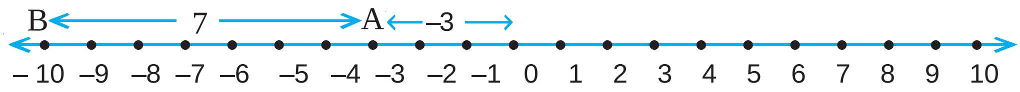
Solution:

(i)



A represents 9 on the number line. Going 5 units to the left of A, we reach the point B, which represents 4.

$\therefore\ 9 + (-5) = 4$

(ii) 

Starting from zero (0) and going three units to the left of zero, we reach the point A, which represents – 3. From A going 7 units to the left of A, we reach the point B which represents – 10.

$\therefore (-3) + (-7) = -10$

1.3 RATIONAL NUMBERS

Consider the situation, when an integer a is divided by another non-zero integer b. The following cases arise:

(i) When 'a' is a multiple of 'b'

Suppose a = mb, where m is a natural number or integer, then $\frac{a}{b} = m$

(ii) When a is not a multiple of b

In this case $\frac{a}{b}$ is not an integer, and hence is a new type of number. Such a number is called a rational number.

Thus, a number which can be put in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$, is called a rational number

Thus, $-\frac{2}{3}, \frac{5}{-8}, \frac{6}{2}, \frac{11}{7}$ are all rational numbers.

1.3.1 Positive and Negative Rational Numbers

(i) A rational number $\frac{p}{q}$ is said to be a positive rational number if p and q are both positive or both negative integers

Thus $\frac{3}{4}, \frac{5}{6}, \frac{-3}{-2}, \frac{-8}{-6}, \frac{-12}{-57}$ are all positive rationals.

(ii) If the integes p and q are of different signs, then $\frac{p}{q}$ is said to be a negaive rational number.

Thus, $\frac{-7}{2}, \frac{6}{-5}, \frac{-12}{4}, \frac{16}{-3}$ are all negaive rationals.

1.3.2 Standard form of a Rational Number

We know that numbers of the form

$$\frac{-p}{q}, \frac{p}{-q}, \frac{-p}{-q} \text{ and } \frac{p}{q}$$

are all rational numbers, where p and q are positive integers

We can see that

$$\frac{-p}{q} = -\left(\frac{p}{q}\right), \frac{-p}{-q} = \frac{-(-p)}{-(-q)} = \frac{p}{q}, \frac{p}{-q} = \frac{(-p)}{-(-q)} = \frac{-p}{q},$$

In each of the above cases, we have made the denominator q as positive.

A rational number $\frac{p}{q}$, where p and q are integers and $q \neq 0$, in which q is positive (or made positive) and p and q are co-prime (i.e. when they do not have a common factor other than 1 and –1) is said to be in standard form.

Thus the standard form of the rational number $\frac{2}{-3}$ is $\frac{-2}{3}$. Similarly, $\frac{-5}{6}$ and $\frac{-3}{5}$ are rational numbers in standard form.

Note: "A rational number in standard form is also referred to as "a rational number in its lowest form". In this lesson, we will be using these two terms interchangably.

For example, rational number $\frac{18}{27}$ can be written as $\frac{2}{3}$ in the standard form (or the lowest form) .

Similarly, $\frac{25}{-35}$, in standard form (or in lowest form) can be written as $\frac{-5}{7}$ (cancelling out 5 from both numerator and denominator).

Some Important Results

(i) Every natural number is a rational number but the vice-versa is not always true.

(ii) Every whole number and integer is a rational number but vice-versa is not always true.

Example 1.5: Which of the following are rational numbers and which are not?

$$-2, \frac{5}{3}, -17, \frac{15}{7}, \frac{18}{5}, -\frac{7}{6}$$

Solution:

(i) –2 can be written as $\frac{-2}{1}$, which is of the form $\frac{p}{q}$, $q \neq 0$. Therefore, –2 is a rational number.

(ii) $\frac{5}{3}$ is a rational number, as it is of the form $\frac{p}{q}$, $q \neq 0$

(iii) –17 is also a rational number as it is of the form $\frac{-17}{1}$

(iv) Similarly, $\frac{15}{7}, \frac{18}{5}$ and $\frac{-7}{6}$ are all rational numbers according to the same argument

Example 1.6: Write the following rational numbers in their lowest terms:

(i) $\frac{-24}{192}$ (ii) $\frac{12}{168}$ (iii) $\frac{-21}{49}$

Solution:

(i) $\frac{-24}{192} = \frac{-3 \times 8}{3 \times 8 \times 8} = \frac{-1}{8}$

$-\frac{1}{8}$ is the lowest form of the rational number $\frac{-24}{192}$

(ii) $\frac{12}{168} = \frac{12}{12 \times 14} = \frac{1}{14}$

$\therefore \frac{1}{14}$ is the lowest form of the rational number $\frac{12}{168}$

(iii) $\frac{-21}{49} = \frac{-3 \times 7}{7 \times 7} = \frac{-3}{7}$

$\therefore \frac{-3}{7}$ is the lowest form of the rational number $\frac{-21}{49}$

1.4 EQUIVALENT FORMS OF A RATIONAL NUMBER

A rational number can be written in an equivalent form by multiplying/dividing the numerator and denominator of the given rational number by the same number.

For example

$$\frac{2}{3}=\frac{2\times 2}{3\times 2}=\frac{4}{6},\quad \frac{2}{3}=\frac{2\times 4}{3\times 4}=\frac{8}{12},\qquad \frac{2\times 8}{3\times 8}=\frac{16}{24}$$

$\therefore \frac{4}{6},\ \frac{8}{12},\ \frac{16}{24}$ etc. are equivalent forms of the rational number $\frac{2}{3}$

Similarly

$$\frac{3}{8}=\frac{6}{16}=\frac{21}{56}=\frac{27}{72}=\ldots$$

and $$\frac{4}{7}=\frac{8}{14}=\frac{12}{21}=\frac{28}{49}=\ldots$$

are equivalent forms of $\frac{3}{8}$ and $\frac{4}{7}$ respectively.

Example 1.7: Write five equivalent forms of the following rational numbers:

(i) $\frac{3}{17}$ (ii) $\frac{-5}{9}$

Solution:

(i) $$\frac{3}{17}=\frac{3\times 2}{17\times 2}=\frac{6}{34},\ \frac{3}{17}=\frac{3\times 4}{17\times 4}=\frac{12}{68},\ \frac{3\times(-3)}{17\times(-3)}=\frac{-9}{-51}$$

$$\frac{3\times 8}{17\times 8}=\frac{24}{136},\ \frac{3}{17}\times\frac{7}{7}=\frac{21}{119}$$

$\therefore$ Five equivalent forms of $\frac{3}{17}$ are

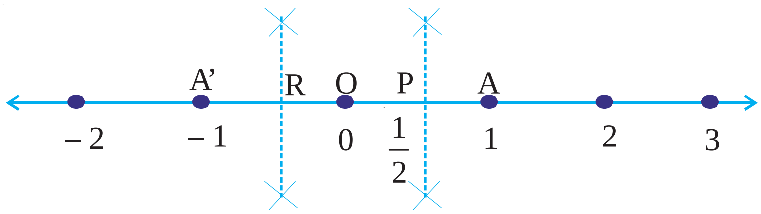
$$\frac{6}{34},\frac{12}{68},\frac{-9}{-51},\frac{24}{136},\frac{21}{119}$$

(ii) As in part (i), five equivalent forms of $\frac{-5}{9}$ are

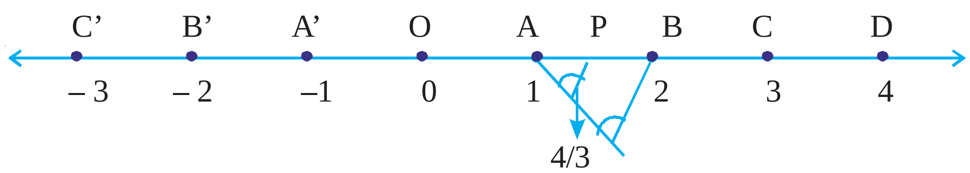
$$\frac{-10}{18}, \frac{-15}{27}, \frac{-20}{36}, \frac{-60}{108}, \frac{-35}{63}$$

1.5 RATIONAL NUMBERS ON THE NUMBER LINE

We know how to represent integers on the number line. Let us try to represent $\frac{1}{2}$ on the number line. The rational number $\frac{1}{2}$ is positive and will be represented to the right of zero. As $0 < \frac{1}{2} < 1$, $\frac{1}{2}$ lies between 0 and 1. Divide the distance OA in two equal parts. This can be done by bisecting OA at P. Let P represent $\frac{1}{2}$. Similarly R, the mid-point of OA', represents the rational number $-\frac{1}{2}$.



Similarly, $\frac{4}{3}$ can be represented on the number line as below:



As $1 < \frac{4}{3} < 2$, therefore $\frac{4}{3}$ lies between 1 and 2. Divide the distance AB in three equal parts. Let one of this part be AP

Now $\frac{4}{3} = 1 + \frac{1}{3} = OA + AP = OP$

The point P represents $\frac{4}{3}$ on the number line.

1.6 COMPARISON OF RATIONAL NUMBERS

In order to compare two rational numbers, we follow any of the following methods:

(i) If two rational numbers, to be compared, have the same denominator, compare their numerators. The number having the greater numerator is the greater rational number.

Thus for the two rational numbers $\frac{5}{17}$ and $\frac{9}{17}$, with the same positive denominator

$17, \frac{9}{17} > \frac{5}{17}$ as $9 > 5$

$\therefore \frac{9}{17} > \frac{5}{17}$

(ii) If two rational numbers are having different denominators, make their denominators equal by taking their equivalent form and then compare the numerators of the resulting rational numbers. The number having a greater numerator is greater rational number.

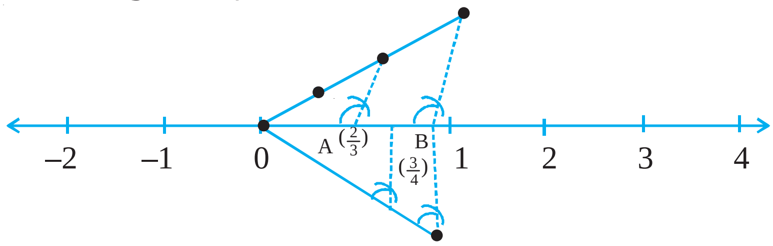
For example, to compare two rational numbers $\frac{3}{7}$ and $\frac{6}{11}$, we first make their denominators same in the following manner:

$$\frac{3\times11}{7\times11} = \frac{33}{77} \text{ and } \frac{9\times7}{11\times7} = \frac{42}{77}$$

As $42 > 33$, $\frac{42}{77} > \frac{33}{77}$ or $\frac{6}{11} > \frac{3}{7}$

(iii) By plotting two given rational numbers on the number line we see that the rational number to the right of the other rational number is greater.

For example, take $\frac{2}{3}$ and $\frac{3}{4}$, we plot these numbers on the number line as below:



$0 < \frac{2}{3} < 1$ and $0 < \frac{3}{4} < 1$. It means $\frac{2}{3}$ and $\frac{3}{4}$ both lie between 0 and 1. By the method of dividing a line into equal number of parts, A represents $\frac{2}{3}$ and B represents $\frac{3}{4}$

As B is to the right of A, $\frac{3}{4} > \frac{2}{3}$ or $\frac{2}{3} < \frac{3}{4}$

$\therefore$ Out of $\frac{2}{3}$ and $\frac{3}{4}$, $\frac{3}{4}$ is the greater number.

CHECK YOUR PROGRESS 1.1

1. Identify rational numbers and integers from the following:

   $4, \frac{-3}{4}, \frac{5}{6}, -36, \frac{12}{7}, \frac{3}{-8}, \frac{15}{7}, -6$

2. From the following identify those which are not :

   (i) natural numbers

   (ii) whole numbers

   (iii) integers

   (iv) rational numbers

   $-\frac{7}{4}, 16, \frac{-3}{7}, -15, 0, \frac{5}{17}, \frac{3}{-4}, -\frac{4}{3}$

3. By making the following rational numbers with same denominator, simplify the following and specify whether the result in each case is a natural number, whole number, integer or a rational number:

   (i) $3+\frac{7}{3}$ (ii) $-3+\frac{10}{4}$ (iii) $-8-13$ (iv) $12-12$

   (v) $\frac{9}{2}-\frac{1}{2}$ (vi) $2\times\frac{5}{7}$ (vii) $8\div 3$

4. Use the number line to add the following:-

   (i) 9 + (–7) (ii) (–5) + (–3) (iii) (–3) + (4)

5. Which of the following are rational numbers in lowest term?

$$\frac{8}{12}, \frac{5}{7}, \frac{-3}{12}, \frac{-6}{7}, \frac{2\sqrt{3}}{\sqrt{27}}, \frac{15}{24}$$

6. Which of the following rational numbers are integers?

$$-10, \frac{15}{5}, \frac{-5}{15}, \frac{13}{5}, \frac{27}{9}, \frac{7\times 3}{14}, \frac{-6}{-2}$$

7. Write 3 rational numbers equivalent to given rational numbers:

$$\frac{2}{5}, \frac{-5}{6}, \frac{17}{3}$$

8. Represent the following rational numbers on the number line.

$$\frac{2}{5}, \frac{3}{4}, \frac{1}{2}$$

9. Compare the following rational numbers by (i) changing them to rational numbers in equivalent forms (ii) using number line:

(a) $\frac{2}{3}$ and $\frac{3}{4}$ (b) $\frac{3}{5}$ and $\frac{7}{9}$ (c) $\frac{-2}{3}$ and $\frac{-1}{2}$

(d) $\frac{3}{7}$ and $\frac{5}{11}$ (e) $\frac{-7}{6}$ and $\frac{3}{2}$

1.7 FOUR FUNDAMENTAL OPERATIONS ON RATIONAL NUMBERS

1.7.1 Addition of Rational Numbers

(a) Consider the addition of rational numbers $\frac{p}{q}, \frac{r}{q}$

$$\frac{p}{q} + \frac{r}{q} = \frac{p+r}{q}$$

For example

(i) $\frac{2}{3} + \frac{5}{3} = \frac{2+5}{3} = \frac{7}{3}$

(ii) $\frac{3}{17} + \frac{9}{17} = \frac{3+9}{17} = \frac{12}{17}$

and (iii) $\frac{14}{3} + \left(\frac{-5}{3}\right) = \frac{14-5}{3} = \frac{9}{3} = 3$

(b) Consider the two rational numbers $\frac{p}{q}$ and $\frac{r}{s}$.

$$\frac{p}{q}+\frac{r}{s}=\frac{ps}{qs}+\frac{rq}{sq}=\frac{ps+rq}{qs}$$

For example,

$$\text{(i)}\ \frac{3}{4}+\frac{2}{3}=\frac{3\times3+4\times2}{4\times3}=\frac{9+8}{12}=\frac{17}{12}$$

$$\text{(ii)}\ -\frac{4}{5}+\frac{7}{8}=\frac{-4\times8+5\times7}{5\times8}=\frac{35-32}{40}=\frac{3}{40}$$

From the above two cases, we generalise the following rule:

(a) The addition of two rational numbers with common denominator is the rational number with common denominator and numerator as the sum of the numerators of the two rational numbers.

(b) The sum of two rational numbers with different denominators is a rational number with the denominator equal to the product of the denominators of two rational numbers and the numerator equal to sum of the product of the numerator of first rational number with the denominator of second and the product of numerator of second rational number and the denominator of the first rational number.

Let us take sone examples:

Example 1.8: Add the following rational numbers:

(i) $\frac{2}{7}$ and $\frac{6}{7}$ (ii) $\frac{4}{17}$ and $\frac{-3}{17}$ (iii) $-\frac{5}{11}$ and $\frac{-3}{11}$

Solution: (i) $\frac{2}{7}+\frac{6}{7}=\frac{2+6}{7}=\frac{8}{7}$

$$\therefore \frac{2}{7}+\frac{6}{7}=\frac{8}{7}$$

(ii) $\frac{4}{17}+\frac{(-3)}{17}=\frac{4+(-3)}{17}=\frac{4-3}{17}=\frac{1}{17}$

$$\therefore \frac{4}{17}+\frac{(-3)}{17}=\frac{1}{17}$$

(iii) $\left(-\frac{5}{11}\right)+\left(\frac{-3}{11}\right)=\frac{(-5)+(-3)}{11}=\frac{-5-3}{11}=\frac{-8}{11}$

$$\therefore \left(-\frac{5}{11}\right)+\left(-\frac{3}{11}\right)=-\frac{8}{11}$$

Example 1.9: Add each of the following rational numbers:

(i) $\frac{3}{4}$ and $\frac{1}{7}$ (ii) $\frac{2}{7}$ and $\frac{3}{5}$ (iii) $\frac{5}{9}$ and $-\frac{4}{15}$

Solution: (i) We have $\frac{3}{4}+\frac{1}{7}$

$$=\frac{3\times 7}{4\times 7}+\frac{1\times 4}{7\times 4}$$

$$=\frac{21}{28}+\frac{4}{28}=\frac{21+4}{28}$$

$$=\frac{25}{28}$$

$$\therefore \frac{3}{4}+\frac{1}{7}=\frac{25}{28} \text{ or } \left[\frac{3\times 7+4\times 1}{4\times 7}=\frac{21+4}{28}=\frac{25}{28}\right]$$

(ii) $\frac{2}{7}+\frac{3}{5}$

$$=\frac{2\times 5}{7\times 5}+\frac{3\times 7}{5\times 7}$$

$$=\frac{10}{35}+\frac{21}{35}$$

$$=\frac{10+21}{35}=\frac{31}{35}$$

$$\therefore \frac{2}{7}+\frac{3}{5}=\frac{31}{35} \text{ or } \left[\frac{2\times 5+3\times 7}{35}=\frac{10+21}{35}=\frac{31}{35}\right]$$

(iii) $\frac{5}{9}+\frac{(-4)}{15}$

$$=\frac{5\times 15}{9\times 15}+\frac{(-4)\times 9}{15\times 9}$$

$$=\frac{75}{135}+\frac{(-36)}{135}$$

$$= \frac{75-36}{135} = \frac{39}{135} = \frac{3\times13}{3\times45} = \frac{13}{45}$$

$$\therefore \frac{5}{9} + \frac{(-4)}{15} = \frac{13}{45} \text{ or } \left[\frac{5\times15+9\times(-4)}{9\times15} = \frac{75-36}{135} = \frac{39}{135} = \frac{13}{45}\right]$$

1.7.2 Subtraction of Rational Numbers

(a) $\frac{p}{q} - \frac{r}{q} = \frac{p-r}{q}$

(b) $\frac{p}{q} - \frac{r}{s} = \frac{ps-qr}{qs}$

Example 1.10: Simplify the following:

(i) $\frac{7}{4} - \frac{1}{4}$ (ii) $\frac{3}{5} - \frac{2}{12}$

Solution: (i) $\frac{7}{4} - \frac{1}{4} = \frac{7-1}{4} = \frac{6}{4} = \frac{2\times3}{2\times2} = \frac{3}{2}$

(ii) $\frac{3}{5} - \frac{2}{12} = \frac{3\times12}{5\times12} - \frac{2\times5}{12\times5}$

$$= \frac{36}{60} - \frac{10}{60} = \frac{36-10}{60}$$

$$= \frac{26}{60} = \frac{13\times2}{30\times2} = \frac{13}{30}$$

1.7.3 Multiplication and Division of Rational Numbers

(i) Multiplication of two rational number $\left(\frac{p}{q}\right)$ and $\left(\frac{r}{s}\right)$, $q \neq 0$, $s \neq 0$ is the rational number $\frac{pr}{ps}$ where $qs \neq 0$

$$= \frac{\text{product of numerators}}{\text{product of denominators}}$$

(ii) Division of two rational numbers $\frac{p}{q}$ and $\frac{r}{s}$, such that $q \neq 0$, $s \neq 0$, is the rational number $\frac{ps}{qr}$, where $qr \neq 0$

In other words $\left(\frac{p}{q}\right) \div \left(\frac{r}{s}\right) = \frac{p}{q} \times \left(\frac{s}{r}\right)$

Or (First rational number) × (Reciprocal of the second rational number)

Let us consider some examples.

Example 1.11: Multiply the following rational numbers:

(i) $\frac{3}{7}$ and $\frac{2}{9}$ (ii) $\frac{5}{6}$ and $\left(\frac{-2}{19}\right)$ (iii) $\frac{7}{13}$ and $\left(\frac{-2}{-5}\right)$

Solution: (i) $\frac{3}{7} \times \frac{2}{9} = \frac{3 \times 2}{7 \times 9} = \frac{3 \times 2}{7 \times 3 \times 3} = \frac{2}{21}$

$$\therefore \left(\frac{3}{7}\right) \times \left(\frac{2}{9}\right) = \frac{2}{21}$$

(ii) $\frac{5}{6} \times \left(\frac{-2}{19}\right) = \frac{5 \times (-2)}{6 \times 19}$

$$= -\frac{2 \times 5}{2 \times 3 \times 19} = -\frac{5}{57}$$

$$\therefore \left(\frac{5}{6}\right) \times \left(-\frac{2}{19}\right) = -\frac{5}{57}$$

(iii) $\frac{7}{13} \times \left(\frac{-2}{-5}\right) = \left(\frac{7}{13}\right)\left(\frac{-(-2)}{5}\right)$

$$= \frac{7}{13} \times \frac{2}{5} = \frac{7 \times 2}{13 \times 5} = \frac{14}{65}$$

$$\therefore \left(\frac{7}{13}\right) \times \left(\frac{-2}{-5}\right) = \frac{14}{65}$$

Example 1.12: Simply the following:

(i) $\left(\frac{3}{4}\right) \div \left(\frac{7}{12}\right)$ (ii) $\frac{9}{16} \div \left(-\frac{105}{12}\right)$ (iii) $\left(\frac{87}{27}\right) \div \left(\frac{29}{18}\right)$

Solution: (i) $\left(\frac{3}{4}\right) \div \left(\frac{7}{12}\right)$

$$= \left(\frac{3}{4}\right) \times \left(\frac{12}{7}\right) \qquad \left[\text{Reciprocal of } \frac{7}{12} \text{ is } \frac{12}{7}\right]$$

$$= \frac{3 \times 12}{4 \times 7} = \frac{3 \times 3 \times 4}{7 \times 4} = \frac{9}{7}$$

$$\therefore \left(\frac{3}{4}\right) \div \left(\frac{7}{12}\right) = \frac{9}{7}$$

(ii) $\left(\frac{9}{16}\right) \div \left(\frac{-105}{2}\right)$

$$\left(\frac{9}{16}\right) \times \left(\frac{2}{-105}\right) \qquad \left[\text{Reciprocal of } \frac{-105}{2} \text{ is } \frac{2}{-105}\right]$$

$$= -\frac{9 \times 2}{2 \times 8 \times 3 \times 35} = -\frac{3 \times 3 \times 2}{2 \times 8 \times 3 \times 35}$$

$$= \frac{-3}{8 \times 35} = \frac{-3}{280}$$

$$\therefore \left(\frac{9}{16}\right) \div \left(\frac{-105}{2}\right) = \frac{-3}{280}$$

(iii) $\left(\frac{87}{27}\right) \div \left(\frac{29}{18}\right)$

$$= \left(\frac{87}{27}\right) \times \left(\frac{18}{29}\right) = \frac{87}{27} \times \frac{18}{29} = \frac{29 \times 3 \times 2 \times 9}{9 \times 3 \times 29} = \frac{2}{1}$$

$$\therefore \left(\frac{87}{27}\right) \div \left(\frac{29}{18}\right) = \frac{2}{1}$$

CHECK YOUR PROGRESS 1.2

1. Add the following rational numbers:

   (i) $\frac{3}{7}, \frac{6}{7}$ (ii) $\frac{2}{15}, -\frac{6}{15}$ (iii) $\frac{3}{20}, \frac{-7}{-20}$ (iv) $\frac{1}{8}, \frac{3}{8}$

2. Add the following rational numbers:

   (i) $\frac{3}{2}, \frac{5}{3}$ (ii) $\frac{17}{7}, \frac{5}{9}$ (iii) $\frac{2}{5}, \frac{-5}{7}$

3. Perform the indicated operations:

   (i) $\left(-\frac{7}{8}+\frac{-5}{12}\right)+\frac{3}{16}$ (ii) $\left(\frac{7}{3}+\frac{3}{4}\right)+\left(-\frac{3}{5}\right)$

4. Subtract:-

   (i) $\frac{7}{15}$ from $\frac{13}{15}$ (ii) $\frac{7}{3}$ from $-\frac{5}{3}$ (iii) $\frac{3}{7}$ from $\frac{9}{24}$

5. Simplify:-

   (i) $\left(3\frac{1}{5}+\frac{7}{5}-2\frac{1}{6}\right)$ (ii) $\frac{5}{2}+\frac{13}{4}-6\frac{3}{4}$

6. Multiply:-

   (i) $\frac{2}{11}$ by $\frac{5}{6}$ (ii) $-\frac{3}{11}$ by $\frac{-33}{35}$ (iii) $\frac{-11}{3}$ by $\frac{-27}{77}$

7. Divide:

   (i) $\frac{1}{2}$ by $\frac{1}{4}$ (ii) $\frac{-7}{4}$ by $\frac{-4}{5}$ (iii) $\frac{35}{33}$ by $\frac{-7}{22}$

8. Simplify the following:

   (i) $\left(\frac{2}{3}+\frac{7}{8}\right)\times\frac{8}{25}\div\frac{37}{15}$ (ii) $\left[\left(\frac{3}{4}-\frac{2}{3}\right)\div\frac{1}{4}\right]\times 21$

9. Divide the sum of $\frac{16}{7}$ and $\frac{-3}{14}$ by their difference.

10. A number when multiplied by $\frac{13}{3}$ gives $\frac{39}{12}$. Find the number.

1.8 DECIMAL REPRESENTATION OF A RATIONAL NUMBER

You are familiar with the division of an integer by another integer and expressing the result as a decimal number. The process of expressing a rational number into decimal form is to carryout the process of long division using decimal notation.

Let us consider some examples.

Example 1.13: Represent each one of the following into a decimal number:

(i) $\frac{12}{5}$ (ii) $\frac{-27}{25}$ (iii) $\frac{13}{16}$

Solution: i) Using long division, we get

$$\begin{array}{r} 2.4 \\ 5\overline{)12.0} \\ \underline{10} \\ 2.0 \\ \underline{2.0} \\ \times \end{array}$$

Hence, $\frac{12}{5} = 2.4$

ii)

$$\begin{array}{r} 25\overline{)-27}\,(-1.08 \\ \underline{25} \\ 200 \\ \underline{200} \\ \times \end{array}$$

Hence, $\frac{-27}{25} = -1.08$

iii)

$$\begin{array}{r} 0.8125 \\ 16\overline{)13.0000} \\ \underline{128} \\ 20 \\ \underline{16} \\ 40 \\ \underline{32} \\ 80 \\ \underline{80} \\ \times \end{array}$$

Hence, $\frac{13}{16} = 0.8125$

From the above examples, it can be seen that the division process stops after a finite number of steps, when the remainder becomes zero and the resulting decimal number has a finite number of decimal places. Such decimals are known as **terminating decimals**.

Note: Note that in the above division, the denominators of the rational numbers had only 2 or 5 or both as the only prime factors.

Alternatively, we could have written $\frac{12}{5}$ as $\frac{12\times 2}{5\times 2} = \frac{24}{10} = 2.4$ and similarly for the others

Let us consider another example.

Example 14: Write the decimal representation of each of the following:

(a) $\frac{7}{3}$ (b) $\frac{2}{7}$ (c) $\frac{5}{11}$

Solution: (a)

$$\begin{array}{r} 2.33 \\ 3\overline{)7.00} \\ 6 \\ \hline 1.0 \\ 9 \\ \hline 1.0 \\ 9 \\ \hline 1.00 \end{array}$$

Here the remainder 1 repeats.

$\therefore$ The decimal is not a terminating decimal

$\frac{7}{3} = 2.333\ldots$ or $2.\bar{3}$

(b)

$$\begin{array}{r} 0.28571428 \\ 7\overline{)2.000} \\ 14 \\ \hline 60 \\ 56 \\ \hline 40 \\ 35 \\ \hline 50 \\ 49 \\ \hline 10 \\ 7 \\ \hline 30 \\ 28 \\ \hline 20 \\ 14 \\ \hline 60 \\ 56 \\ \hline 4 \end{array}$$

Here when the remainder is 3, the digit after that start repeating

$\frac{2}{7} = 0.\overline{285714}$

Note: A bar over a digit or a group of digits implies that digit or that group of digits starts repeating itself indefinitely.

(c)

$$\begin{array}{r} 0.454 \\ 11\overline{)5.00} \\ 44 \\ \hline 60 \\ 55 \\ \hline 50 \\ 44 \\ \hline 50\ldots \end{array}$$

Here again when the remainder is 5, the digits after 5 start repeating

$\therefore \frac{5}{11} = 0.\overline{45}$

From the above, it is clear that in cases where the denominator has factors other than 2 or 5, the decimal representation starts repeating. Such decimals are called non-terminating repeating decimals.

Thus, we see from examples 1.13 and 1.14 that the decimal representation of a rational number is

(i) either a terminating decimal (and the remainder is zero after a finite number of steps)

(ii) or a non-terminating repeating decimal (where the division will never end)

∴ Thus, a rational number is either a terminating decimal or a non-terminating repeating decimal

1.8 EXPRESSING DECIMAL EXPANSION OF A RATIONAL NUMBER IN p/q FORM

Let us explain it through examples

Example 1.15: Express (i) 0.48 and (ii) 0.1357 in $\frac{p}{q}$ form

Solution: (i) $0.48 = \frac{48}{100} = \frac{12}{25}$

(ii) $0.1375 = \frac{1375}{10000} = \frac{55}{400} = \frac{11}{80}$

Example 1.16: Express (i) 0.666... (ii) 0.374374... in $\frac{p}{q}$ form

Solution: (i) Let $x = 0.666...$ (A)

$\therefore\ 10\,x = 6.666...$ (B)

(B) – (A) gives $9\,x = 6$ or $x = \frac{2}{3}$

(ii) Let $x = 0.374374374....$ (A)

$1000\,x = 374.374374374....$ (B)

(B) – (A) gives $999\,x = 374$

or $x = \frac{374}{999}$

$$\therefore 0.374374374... = \frac{374}{999}$$

The above example illustrates that:

A terminating decimal or a non-terminating recurring decimal represents a rational number

Note: The non- terminating recurring decimals like 0.374374374... are written as $0.\overline{374}$. The bar on the group of digits 374 indicate that the group of digits repeats again and again.

CHECK YOUR PROGRESS 1.3

1. Represent the following rational numbers in the decimal form:

   (i) $\frac{31}{80}$ (ii) $\frac{12}{25}$ (iii) $\frac{12}{8}$ (iv) $\frac{75}{12}$ (v) $\frac{91}{63}$

2. Represent the following rational numbers in the decimal form:

   (i) $\frac{2}{3}$ (ii) $\frac{5}{7}$ (iii) $\frac{25}{11}$

3. Represent the following decimals in the form $\frac{p}{q}$.

   (a) (i) 2.3 (ii) – 3.12 (iii) –0.715 (iv) 8.146

   (b) (i) $0.\overline{333}$ (ii) $3.\overline{42}$ (iii) – 0.315315315...

1.9 RATIONAL NUMBERS BETWEEN TWO RATIONAL NUMBERS

Is it possible to find a rational number between two given rational numbers. To explore this, consider the following examples.

Example 1.17: Find a rational number between $\frac{3}{4}$ and $\frac{6}{5}$

Solution: Let us try to find the number $\frac{1}{2}\left(\frac{3}{4}+\frac{6}{5}\right)$

$$= \frac{1}{2}\left(\frac{15+24}{20}\right) = \frac{39}{40}$$

Now $\quad \frac{3}{4} = \frac{3\times10}{4\times10} = \frac{30}{40}$

and $\quad \frac{6}{5} = \frac{6\times8}{5\times8} = \frac{48}{40}$

Obviously $\quad \frac{30}{40} < \frac{39}{40} < \frac{48}{40}$

i.e. $\frac{39}{40}$ is a rational number between the rational numbers $\frac{3}{4}$ and $\frac{6}{5}$.

Note: $\frac{3}{4} = 0.75,\quad \frac{39}{40} = 0.975 \quad$ and $\quad \frac{6}{5} = 1.2$

$\therefore 0.75 < 0.975 < 1.2$

or $\quad \frac{3}{4} < \frac{39}{40} < \frac{6}{5}$

$\therefore$ This can be done by either way:

(i) reducing each of the given rational number with a common base and then taking their average

or (ii) by finding the decimal expansions of the two given rational numbers and then taking their average.

The question now arises, "How many rationals can be found between two given rationals? Consider the following examples.

Example 1.18: Find 3 rational numbers between $\frac{1}{2}$ and $\frac{3}{4}$.

Solution: $\quad \frac{1}{2} = \frac{1\times8}{2\times8} = \frac{8}{16}$

and $\quad \frac{3}{4} = \frac{3\times4}{4\times4} = \frac{12}{16}$

As $\quad \frac{8}{16} < \frac{9}{16} < \frac{10}{16} < \frac{11}{16} < \frac{12}{16}$

$\therefore$ We have been able to find 3 rational numbers

$$\frac{9}{16}, \frac{10}{16} \text{ and } \frac{11}{16} \text{ between } \frac{1}{2} \text{ and } \frac{3}{4}$$

In fact, we can find any number of rationals between two given numbers.

Again $\frac{1}{2} = \frac{50}{2 \times 50} = \frac{50}{100}$

$$\frac{3}{4} = \frac{3 \times 25}{4 \times 25} = \frac{75}{100}$$

As $\frac{50}{100} < \frac{51}{100} < \frac{52}{100} < \frac{53}{100} < \ldots\ldots < \frac{72}{100} < \frac{73}{100} < \frac{74}{100} < \frac{75}{100} < \ldots\ldots$ (i)

$\therefore$ we have been able to find 24 rational numbers between $\frac{1}{2}$ and $\frac{3}{4}$ as given in (i) above.

We can continue in this way further.

Note: From the above it is clear that between any two rationals an infinite number of rationals can be found.

CHECK YOUR PROGRESS 1.4

1. Find a rational number between the following rational numbers:

   (i) $\frac{3}{4}$ and $\frac{4}{3}$ (ii) 5 and 6 (iii) $-\frac{3}{4}$ and $\frac{1}{3}$

2. Find two rational numbers between the following rational numbers:

   (i) $-\frac{2}{3}$ and $\frac{1}{2}$ (ii) $-\frac{2}{3}$ and $-\frac{1}{4}$

3. Find 5 rational numbers between the following rational numbers:

   (i) 0.27 and 0.30 (ii) 7.31 and 7.35

   (iii) 20.75 and 26.80 (iv) 1.001 and 1.002

1.10 IRRATIONAL NUMBERS

We have seen that the decimal expansion of a rational number is either terminating or is a non-terminating and repeating deimal.

Are there decimals which are neither terminating nor non-terminating but repeating decimals? Consider the following decimal:

0.10 100 1000 10000 1....... (i)

You can see that this decimal has a definite pattern and it can be written indefinitely, and there is no block of digits which is repeating. Thus, it is an example of a non-terminating and non-repeating decimal. A similar decimal is given as under:

0.1 2 3 4 5 6 7 8 9 10 11 12 13..... (ii)

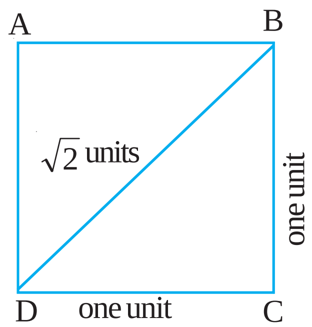
Can you write the next group of digits in (i) and (ii)? The next six digits in (i) are 000001... and in (ii) they are 14 15 16 ...

Such decimals as in (i) and (ii) represent irrational numbers.

Thus, *a decimal expansion which is neither terminating nor is repeating represents an irrational number.*

1.11 INADEQUACY OF RATIONAL NUMBERS

Can we measure all the lengths in terms of rational numbers? Can we measure all weights in terms of rational numbers?



Let us examine the following situation:

Consider a square ABCD, each of whose sides is 1 unit. Naturally the diagonal BD is of length $\sqrt{2}$ units.

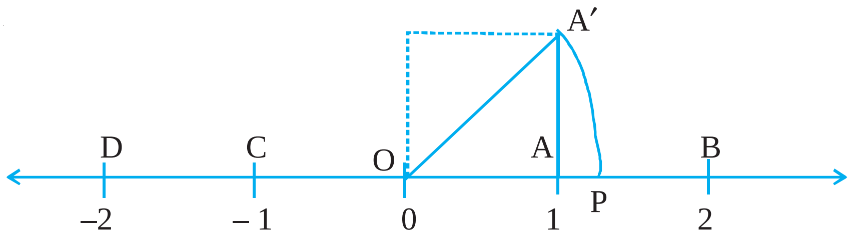
It can be proved that $\sqrt{2}$ is not a rational number, as there is no rational, whose square is 2, [Proof is beyond the scope of this lesson].

We conclude that we can not exactly measure the lengths of all line-segments using rationals, in terms of a given unit of length. Thus, the rational numbers are inadequate to measure all lengths in terms of a given unit. This inadequacy necessitates the extension of rational numbers to irrationals (which are not rational)

We have also read that corresponding to every rational number, there corresponds a point on the number line. Consider the converse of this statement:

Given a point on the number line, will it always correspond to a rational number? The answer to this question is also "No". For clarifying this, we take the following example.

On the number line take points O, A, B, C and D representing rational 0, 1, 2, –1 and –2 respectively. At A draw $AA' \perp$ to OA such that $AA' = 1$ unit



$\therefore OA' = \sqrt{1^2 + 1^2} = \sqrt{2}$ units. Taking O as centre and radius OA′, if we draw an arc, we reach the point P, which represents the number $\sqrt{2}$.

As $\sqrt{2}$ is irrational, we conclude that there are points on the number line (like P) which are not represented by a rational number. Similarly, we can show that we can have points like $\sqrt{3},\ 2\sqrt{3},\ 5\sqrt{2}$ etc, which are not represented by rationals.

$\therefore$ The number line, consisting of points corresponding to rational numbers, has gaps on it. Therefore, the number line consists of points corresponding to rational numbers and irrational numbers both.

We have thus extended the system of rational numbers to include irrational numbers also. The system containing rationals and irrationals both is called the Real Number System.

The system of numbers consisting of all rational and irrational numbers is called the system of real numbers.

CHECK YOUR PROGRESS 1.5

1. Write the first three digits of the decimal representation of the following:

   $\sqrt{2},\ \sqrt{3},\ \sqrt{5}$

2. Represent the following numbers on the real number line:

   (i) $\frac{\sqrt{2}}{2}$ (ii) $1+\sqrt{2}$ (iii) $\frac{\sqrt{3}}{2}$

1.12 FINDING IRRATIONAL NUMBER BETWEEN TWO GIVEN NUMBERS

Let us illustrate the process of finding an irrational number between two given numbers with the help of examples.

Example 1.19: Find an irrational number between 2 and 3.

Solution: Consider the number $\sqrt{2 \times 3}$

We know that $\sqrt{6}$ approximately equals 2.45.

$\therefore$ It lies between 2 and 3 and it is an irrational number.

Example 1.20: Find an irrational number lying between $\sqrt{3}$ and 2.

Solution: Consider the number $\dfrac{\sqrt{3}+2}{2}$

$$= 1 + \frac{\sqrt{3}}{2} \approx 1 + \frac{1.732}{2} = 1.866$$

$\therefore \quad \dfrac{\sqrt{3}+2}{2} \approx 1.866$ lies betwen $\sqrt{3}$ (≈ 1.732) and 2

$\therefore$ The required irrational number is $\dfrac{\sqrt{3}+2}{2}$

CHECK YOUR PROGRESS 1.6

1. Find an irrational number between the following pairs of numbers

   (i) 2 and 4 (ii) $\sqrt{3}$ and 3 (iii) $\sqrt{2}$ and $\sqrt{3}$

2. Can you state the number of irrationals between 1 and 2? Illustrate with three examples.

1.13 ROUNDING OFF NUMBERS TO A GIVEN NUMBER OF DECIMAL PLACES

It is sometimes convenient to write the approximate value of a real number upto a desired number of decimal places. Let us illustrate it by examples.

Example 1.21: Express 2.71832 approximately by rounding it off to two places of decimals.

Solution: We look up at the third place after the decimal point. In this case it 8, which is more than 5. So the approximate value of 2.71832, upto two places of decimal is 2.72.

Example 1.22: Find the approximate value of 12.78962 correct upto 3 places of decimals.

Solution: The fourth place of decimals is 6 (more than 5) so we add 1 to the third place to get the approximate value of 12.78962 correct upto three places of decimals as 12.790.

Thus, we observe that to round off a number to some given number of places, we observe the next digit in the decimal part of the number and proceed as below

(i) If the digit is less than 5, we ignore it and state the answer without it.

(ii) If the digit is 5 or more than 5, we add 1 to the preceeding digit to get the required number upto desired number of decimal places.

CHECK YOUR PROGRESS 1.7

1. Write the approximate value of the following correct upto 3 place of decimals.

   (i) 0.77777 (ii) 7.3259 (iii) 1.0118

   (iv) 3.1428 (v) 1.1413

LET US SUM UP

- Recall of natural numbers, whole numbers, integers with four fundamental operations is done.
- Representation of above on the number line.
- Extension of integers to rational numbers - A rational number is a number which can be put in the form p/q, where p and q are integers and $q \neq 0$.
- When q is made positive and p and q have no other common factor, then a rational number is said to be in standard form or lowest form.
- Two rational numbers are said to be the equivalent form of the number if standard forms of the two are same.
- The rational numbers can be represented on the number line.
- Corresponding to a rational number, there exists a unique point on the number line.
- The rational numbers can be compared by
  - reducing them with the same denominator and comparing their numerators.
  - when represented on the number line, the greater rational number lies to the right of the other.

- As in integers, four fundamental operations can be performed on rational numbers also.
- The decimal representation of a rational number is either terminating or non-terminating repeating.
- There exist infinitely many rational numbers between two rational numbers.
- There are points other than those representing rationals on the number line. That shows inadequacy of system of rational numbers.
- The sytem of rational numbers is extended to real numbers.
- Rationals and irrationals together constitute the system of real numbers.
- We can always find an irrational number between two given numbers.
- The decimal representation of an irrational number is non-terminating non repeating.
- We can find the approximate value of a rational or an irrational number upto a given number of decimals.

TERMINAL EXERCISE

1. From the following pick out:

   (i) natural numbers

   (ii) integers which are not natural numbers

   (iii) rationals which are not natural numbers

   (iv) irrational numbers

$$-3, 17, \frac{6}{7}, \frac{-3}{8}, 0, -32, \frac{3}{14}, \frac{11}{6}, \sqrt{2}, 2+\sqrt{3}$$

2. Write the following integers as rational numbers:

   (i) – 14 (ii) 13 (iii) 0 (iv) 2

   (v) 1 (vi) –1 (vii) –25

3. Express the following rationals in lowest terms:

$$\frac{6}{8}, \frac{14}{21}, \frac{-17}{153}, \frac{13}{273}$$

4. Express the following rationals in decimal form:

   (i) $\frac{11}{80}$ (ii) $\frac{8}{25}$ (iii) $\frac{14}{8}$ (iv) $\frac{15}{6}$ (v) $\frac{98}{35}$

(vi) $\frac{15}{7}$ (vii) $-\frac{7}{6}$ (viii) $\frac{115}{11}$ (ix) $-\frac{17}{13}$ (x) $\frac{126}{36}$

5. Represent the following decimals in $\frac{p}{q}$ form:

(i) 2.4 (ii) – 0.32 (iii) 8.14 (iv) $3.\overline{24}$

(v) 0.415415415...

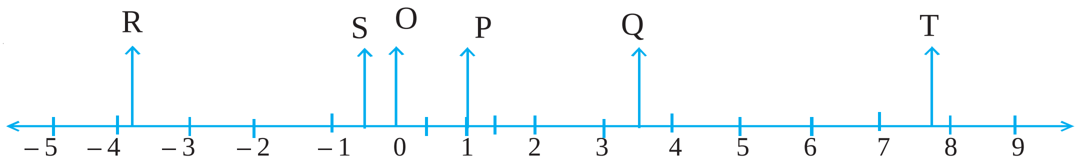
6. Find a rational number betwen the following rational numbers:

(i) $\frac{3}{4}$ and $\frac{7}{8}$ (ii) – 2 and – 3 (iii) $-\frac{4}{5}$ and $\frac{1}{3}$

7. Find three rational numbers between the following rational numbers:

(i) $\frac{3}{4}$ and $\frac{-3}{4}$ (ii) 0.27 and 0.28 (iii) 1.32 and 1.34

8. Write the rational numbers corresponding to the points O, P, Q, R, S and T on the number line in the following figure:



9. Find the sum of the following rational numbers:

(i) $\frac{3}{5}, \frac{-7}{5}$ (ii) $-\frac{7}{9}, \frac{5}{9}$ (iii) $\frac{3}{5}, \frac{7}{3}$ (iv) $\frac{9}{5}, \frac{2}{3}$ (v) $\frac{18}{7}, -\frac{7}{6}$

10. Find the product of the following rationals:

(i) $\frac{3}{5}, \frac{7}{3}$ (ii) $\frac{19}{5}, \frac{2}{3}$ (iii) $\frac{15}{7}, \frac{-14}{5}$

11. Write an irrational number between the following pairs of numbers:

(i) 1 and 3 (ii) $\sqrt{3}$ and 3 (iii) $\sqrt{2}$ and $\sqrt{5}$ (iv) $-\sqrt{2}$ and $\sqrt{2}$

12. How many rational numbers and irrational numbers lie between the numbers 2 and 7?

13. Find the approximate value of the following numbers correct to 2 places of decimals:

(i) 0.338 (ii) 3.924 (iii) 3.14159 (iv) 3.1428

14. Write the value of following correct upto 3 places of decimals:

(i) $\frac{3}{4}$ (ii) $2+\sqrt{2}$ (iii) 1.7326 (iv) 0.9999...

15. Simplify the following as irrational numbers. The first one is done for you.

(i) $12\sqrt{3}+5\sqrt{3}-7\sqrt{3}=\sqrt{3}[12+5-7]=10\sqrt{3}$

(ii) $3\sqrt{2}-2\sqrt{8}+7\sqrt{2}$

(iii) $3\sqrt{2}\times 2\sqrt{3}\times 5\sqrt{6}$

(iv) $[(\sqrt{8}\times 3\sqrt{2})\times 6\sqrt{2}]\div 36\sqrt{2}$

ANSWERS TO CHECK YOUR PROGRESS

1.1

1. Integers: 4, – 36, – 6

   Rational Numbers: $4,\frac{-3}{4},\frac{5}{6},-36,\frac{12}{7},\frac{-3}{8},\frac{15}{7},-6$

2. (i) $-\frac{7}{4},-\frac{3}{7},-15,0.\frac{5}{17},-\frac{3}{4},-\frac{4}{3}$

   (ii) $-\frac{7}{4},-\frac{3}{7},-15,\frac{5}{17},-\frac{3}{4},-\frac{4}{3}$

   (iii) $-\frac{7}{4},-\frac{3}{7},\frac{5}{17},-\frac{3}{4},-\frac{4}{3}$

   (iv) All are rational numbers.

3. (i) $\frac{16}{3}$, rational (ii) $-\frac{1}{2}$, rational (iii) –21, integer and rational

   (iv) zero, whole number, integer and rational (v) 4, All

   (vi) $\frac{10}{7}$, rational (vii) $\frac{8}{3}$, rational

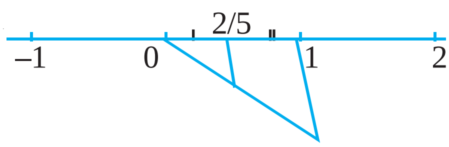
4. (i) 2 (ii) – 8 (iii) 1

5. $\frac{5}{7}, \frac{-6}{7}, \frac{2\sqrt{3}}{\sqrt{27}}$

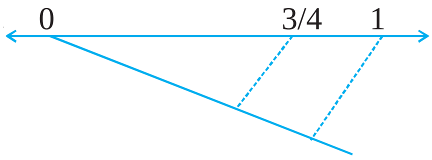
6. $-10, \frac{15}{5}, \frac{27}{9}, \frac{-6}{-2}$

7. (i) $\frac{2}{5} = \frac{4}{10} = \frac{6}{15} = \frac{8}{20}$ (ii) $-\frac{5}{6} = -\frac{10}{12} = -\frac{15}{18} = -\frac{20}{24}$ (iii) $\frac{17}{3} = \frac{34}{6} = \frac{51}{9} = \frac{68}{12}$

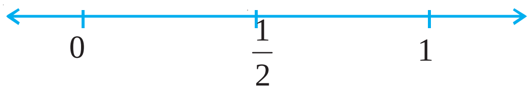
8. (i)



(ii)



(iii)



9. (a) $\frac{3}{4} > \frac{2}{3}$ (b) $\frac{7}{9} > \frac{3}{5}$ (c) $\frac{-1}{2} > \frac{-2}{3}$ (d) $\frac{5}{11} > \frac{3}{7}$

(e) $\frac{3}{2} > -\frac{7}{6}$

1.2

1. (i) $\frac{9}{7}$ (ii) $-\frac{4}{15}$ (iii) $\frac{1}{2}$ (iv) $\frac{1}{2}$

2. (i) $\frac{19}{6}$ (ii) $\frac{188}{63}$ (iii) $-\frac{11}{35}$

3. (i) $-\frac{53}{48}$ (ii) $\frac{149}{60}$

4. (i) $\frac{2}{5}$ (ii) -4 (iii) $\frac{-3}{56}$

5. (i) $\frac{73}{30}$ (ii) -1

6. (i) $\frac{5}{33}$ (ii) $\frac{9}{35}$ (iii) $\frac{9}{7}$

7. (i) 2 (ii) $\frac{35}{16}$ (iii) $-\frac{10}{3}$

8. (i) $\frac{1}{5}$ (ii) 7

9. $\frac{29}{35}$

10. $\frac{3}{4}$

1.3

1. (i) 0.3875 (ii) 0.48 (iii) 1.5 (iv) 6.25 (v) $1.\overline{4}$

2. (i) $0.\overline{6}$ (ii) $0.\overline{714285}$ (iii) $2.\overline{27}$

3. (a) (i) $\frac{23}{10}$ (ii) $-\frac{78}{25}$ (iii) $-\frac{143}{200}$ (iv) $\frac{4073}{500}$

   (b) (i) $\frac{1}{3}$ (ii) $\frac{113}{33}$ (iii) $-\frac{35}{111}$

1.4

1. (i) $\frac{25}{24}$ (ii) 5.5 (iii) $\frac{-5}{24}$

2. (i) 0.2 and 0.3 (ii) – 0.30, – 0.35

3. (i) 0.271, 0.275, 0, 281, 0.285, 0.291

   (ii) 7.315, 7.320 7.325, 7.330, 7.331

   (iii) 21.75, 22.75, 23.75, 24.75, 25.75

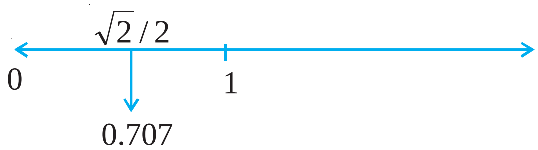
   (iv) 1.0011, 1.0012, 1.0013, 1.0014, 1.0015

   Note: *Can be other answers as well.*

1.5

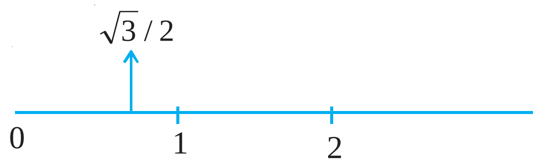
1. 1.414, 1.732, 2.236

2. (i)



(ii)

(iii)



1.6

1. (i) $\sqrt{5}$ (ii) $\sqrt{3}+1$ (iii) $\dfrac{\sqrt{2}+\sqrt{3}}{2}$

2. Infinitely many:

   1.0001, 1.0002, ..., 1.0010, 1.0011,....., 1.0020, 1.0021, ...

1.7

1. (i) 0.778 (ii) 7.326 (iii) 1.012 (iv) 3.143 (v) 1.141

ANASWERS TO TERMINAL EXERCISE

1. Natural: 17,

   Integers but not natural numbers, –3, 0, – 32

   Rationals but not natural numbers: $-3, \dfrac{6}{7}, \dfrac{-3}{8}, 0, -32, \dfrac{3}{14}, \dfrac{11}{6}$

   Irrationals but not rationals: $\sqrt{2}$, $2+\sqrt{3}$

2. (i) $-\dfrac{14}{1}$ (ii) $\dfrac{13}{1}$ (iii) $\dfrac{0}{1}$ (iv) $\dfrac{2}{1}$

   (v) $\dfrac{1}{1}$ (vi) $\dfrac{-1}{1}$ (vii) $\dfrac{-25}{1}$

3. $\dfrac{3}{4}, \dfrac{2}{3}, -\dfrac{1}{9}, \dfrac{1}{21}$

4. (i) 0.1375 (ii) 0.32 (iii) 1.75 (iv) 2.5 (v) 2.8
   (vi) 2.142857 (vii) $-1.\overline{166}$ (viii) $10.\overline{45}$ (ix) $-1.\overline{307692}$ (x) 3.5

5. (i) $\frac{12}{5}$ (ii) $\frac{-8}{25}$ (iii) $\frac{407}{50}$ (iv) $\frac{107}{33}$ (v) $\frac{415}{999}$

6. (i) $\frac{13}{16}$ (ii) -2.5 (iii) zero

7. (i) 0.50, 0.25, 0.00 (ii) 0.271, 0.274, 0.277 (iii) 1.325. 1.33, 1.335

8. (i) R: -3.8 (ii) S: -0.5 (iii) O: 0.00 (iv) S: $-0.\overline{33}$ (v) Q: 3.5
   (vi) T: $7.\overline{66}$

9. (i) $-\frac{4}{5}$ (ii) $-\frac{2}{9}$ (iii) $\frac{44}{15}$ (iv) $\frac{37}{15}$ (v) $\frac{59}{42}$

10. (i) $\frac{7}{5}$ (ii) $\frac{38}{15}$ (iii) -6

11. (i) $\sqrt{3}$ (ii) $1+\sqrt{3}$ (iii) $\sqrt{3}$ (iv) $\frac{\sqrt{2}}{2}$

12. Infinitely many

13. (i) 0.34 (ii) 3.92 (iii) 3.14 (iv) 3.14

14. (i) 0.75 (ii) 3.414 (iii) 1.733 (iv) 1.000

15. (ii) $6\sqrt{2}$ (iii) 180 (iv) 2